“Chain store against manufacturers: regulation can mitigate market distortion”

May 3, 2016

by Igor Bykadorov, Andrea Ellero, Stefania Funari, Sergey Kokovin, and Marina Pudova

(On-line Appendix, May 2016)

This online Appendix contains Intro with the literature review and detailed proofs for this paper, in the order of formulae derivation.

1 Intro and literature review

Emerging domination of chain-stores like Wal-Mart in retailing has rised some controversial questions for consumers and regulating authorities. A political struggle against chain-stores resulted in some restrictions on their construction or/and operations in US and Europe. Similarly, the Russian Retailing law (2010) restricts each store's share in a city district. It also forbids an entrance fee, previously required by retailers from manufacturers. Can such measures be really welfare-enhancing, as proclaimed by the politicians?

To sharpen this question, we model the dominant market position of a big retailer—by a limiting case where only *one* monopolistic retailer faces *numerous* manufacturers (in contrast with some papers considering a single manufacturer facing many retailers). In the present paper, alike our previous work [1]-[3], the retailer plays as a leader against her followers – a continuum of suppliers. They compete in a monopolistically competitive sector with free entry, that involves endogeneous diversity of goods, as in [7], but the representative consumer has a quasilinear quadratic preferences for a differentiated good, as in [11]. The retailer announces her markup before the suppliers choose their prices. They observe the markup and the current level of competition (price-index), correctly anticipating the demand but ignoring the (negligible) influence on each other. The retailer correctly anticipates the equilibrium and also can restrict the mass of manufacturers entering the industry (if finds such restriction profitable). It is a sort of two-tier monopoly, because each supplier practices monopolistic pricing on her variety of the good and the retailer's markup is added to the supplier's markup. Related market distortion can be a reason for some governmental regulation.

The first research question is a *sales tax:* does it soften or enforce the market imperfections arising in such two-tier monopoly? We start with finding a closed-form characterization of equilibria and welfare in our market and comparing these to the socially-optimal firm size (output) and diversity (mass of firms). Rather naturally, in the absence of regulation, the direction of market distortion turns out two-fold: the firms tend to be too *small* and the diversity is also *insufficient* (see the explanation after Lemma 1 and Proposition 1). Thus, monopolism supresses the market in both dimensions, so, there is a room for market regulation. Implementing this idea, we first find a welfare-maximizing tax level without any optimization of the entrance fee. More or less expectedly, the socially-optimal sales tax rate is negative, that means *sales subsidization.* The explanation suggested after Proposition 1 exploits the arguments typical for regulated monopoly but also involves new considerations of optimal diversity.

The second research question concerns an entrance fee, imposed by a retailer onto her suppliers. Such practice was very common in Russia until the 2010 law on retailing had forbidden the (explicit) fee. However, the retailers responded by imposing an indirect entrance fee in the form of obligatory advertising and regular cheap-sales campaigns, required by a retailer from a manufacturer. So, this practice continues, showing its importance for the market. Is it welfare-enhancing or deteriorating, as the legislators supposed? The subsequent explanation emphasizes arguments in favour of two-part tariffs, usual for any monopoly. They remain true in our case, in spite of complications with product diversity and monopolistic competition, unusual for IO. Two-part tariff is a pricing tool that enables the stronger partner in vertical relations (alike many other situations) to *appropriate a potential welfare loss of the economy* as one's profit, simultaneously capturing part of other player's benefits. This tool "integrates the industry" to eliminate natural losses from non-cooperative vertical relations. The delicate detail, specific for our case, is that socially-optimal product diversity (see [7]) does not contradict this general market logic neither in the question of entrance fees, nor in the question of subsidization.

Comparing our findings to the literature on manufacturer-retailer interactions, we can cite Spengler [13], which shows that the retail price is higher in the decentralized structures than in the integrated ones, whereas vertical integration may expand welfare through the elimination of "double marginalization." More recent studies add product differentiation to such modelling and show various effects. In some cases, industry integration is justified because it increases social welfare optimizing the number of retailers (as in Dixit [8]). In other cases such integration deteriorates social welfare, as in Perry and Groff [12] that used the model by Chen [4] (a manufacturer monopolist signs a contract with each retailer; the number of goods is less than the social optimum). Among the reviews on these topics, focuses on the theory of vertical integration, [14] reports the contributions from the theory of organizations and strategic interactions, whereas [9] provides several game-theory models applied to distribution channels. Typically for these models, the manufacturer is the leader of the supply chain and moves first, whereas the retailer is the follower that decides her strategy after observing the choice made by the leader-supplier, while our timing (and distribution of power) is the opposite.

In order to deal with relatively recent retailing phenomenon of dominating giants like Wal-Mart, Achan, Costco, Target—several studies have modelled the vertical distribution channel with retailer's leadership, as we do, although in different contexts and using different industry models. In particular, Choi [5] consideres a monopolistic retailer that sells two competing brands of two manufacturers. The effects on retail prices and social welfare strongly depend on the characteristics of the demand function. Social welfare is enhanced by leadership in case of nonlinear demand. Kadiyali et al. [10] estimate the parameters of a general model that can account for a variety of possible pricing games, including Nash behavior, manufacturer as a leader, and manufacturer as a follower. Xie and Neyret [15] assumed a dominant behavior of the retailer in a game of vertical cooperative advertising, where the manufacturers maintain a certain part of the retailers advertising expenditure. More closely to our theme, Bykadorov et al. [2], [3] study a model like in this paper and compares the equilibrium with retailer-leader to the market equilibrium with other scenarios: the retailer as the follower or "myopic" behavior of Nash equilibrium: the main result is that market concentration (leadership) has a positive effect on welfare. To the best of our knowledge, the questions of this paper: optimal fiscal policy and entrance fee, are not addressed in this literature.

2 Model

2.1 Consumers

The game timing is Retailer - Manufacturers - Consumers. The equilibria derivation by backward induction starts with a consumer program:

$$U(\mathbf{q}, N, m) = \alpha \cdot \int_0^N q(i)\, di - \frac{\beta - \gamma}{2} \cdot \int_0^N (q(i))^2\, di - \frac{\gamma}{2} \cdot \left(\int_0^N q(i)\, di \right)^2 + m \longrightarrow \max_{\mathbf{q}}, \ \ \textbf{s.t.}$$

$$\int_0^N \breve{p}(i)\, q(i)\, di + m \leq L + \int_0^N \pi_{\mathcal{M}}(i)\, di + \pi_{\mathcal{R}},$$

where

$$\breve{p}(i) = p(i) + r(i) + \tau.$$

FOC is:

$$\frac{\partial U}{\partial q(i)} = \alpha - (\beta - \gamma) \cdot q(i) - \gamma \cdot \int_0^N q(j)\, dj = \breve{p}(i)\,. \tag{1}$$

Hence

$$\int_0^N \left(\alpha - (\beta - \gamma) \cdot q(i) - \gamma \int_0^N q(j)\, dj \right) di = \int_0^N \breve{p}(i)\, di,$$

i.e.,

$$N \cdot \left(\alpha - \gamma \cdot \int_0^N q(j)\, dj \right) - (\beta - \gamma) \cdot \int_0^N q(j)\, dj = \int_0^N \breve{p}(j)\, dj,$$

i.e.,

$$\int_0^N q(j)\, dj = \frac{N \cdot \alpha}{\beta - \gamma + \gamma \cdot N} - \frac{1}{\beta - \gamma + \gamma \cdot N} \cdot \int_0^N \breve{p}(j) dj.$$

Now substitute $\int_0^N q(j)dj$ into (1) and find demand as a function of retailer's markup r and mill price p, under price index P:

$$\hat{q}(i, p(i), r(i), P, N) =$$

$$= \frac{\alpha}{\beta - \gamma + \gamma \cdot N} - \frac{1}{\beta - \gamma} \cdot \breve{p}(i) + \frac{\gamma}{(\beta - \gamma) \cdot (\beta - \gamma + \gamma \cdot N)} \cdot \int_0^N \breve{p}(j) dj =$$

$$= a - \frac{1}{\beta - \gamma} \cdot \breve{p}(i) + b \cdot P,$$

where

$$a = \frac{\alpha}{\beta - \gamma + \gamma \cdot N}\,, \qquad b = \frac{\gamma}{(\beta - \gamma) \cdot (\beta - \gamma + \gamma \cdot N)},$$

$$P = \int_0^N \left[p(j) + r(j) + \tau\right] dj.$$

2.2 Manufacturers

Each i-th seller's program is

$$\pi_{\mathcal{M}}(i, p, r(i), P) = (p(i) - c) \cdot \hat{q}(i, p(i), r(i), P, N) - (f_{\mathcal{M}} + f_E) \;\rightarrow\; \max_{p(i)},$$

where

$$\hat{q}(i, p(i), r(i), P, N) = a - \frac{1}{\beta - \gamma} \cdot (p(i) + r(i) + \tau) + b \cdot P.$$

FOC is:

$$\frac{\partial}{\partial p(i)} \pi_{\mathcal{M}}(i, p(i), r(i), P) =$$

$$= \hat{q}(i, p(i), r(i), P, N) + (p(i) - c) \cdot \frac{\partial}{\partial p(i)} \hat{q}(i, p(i), r(i), P, N) =$$

$$= a - \frac{1}{\beta - \gamma} \cdot (p(i) + r(i) + \tau) + b \cdot P - \frac{1}{\beta - \gamma} \cdot (p(i) - c) = 0,$$

i.e.,

$$p^*(i, r(i), P, N) = 0.5 \cdot ((\beta - \gamma) \cdot (a + b \cdot P) - r(i) - \tau + c).$$

In symmetric case we obtain the manufacturer's response to r, N, P magnitudes:

$$p^*(r, P, N) = 0.5 \cdot ((\beta - \gamma) \cdot (a + b \cdot P) - r - \tau + c),$$

$$q^*(r, P, N) = \hat{q}(p^*(r, P, N), r, P, N) = a - \frac{1}{\beta - \gamma} \cdot (p^*(r, P, N) + r + \tau) + b \cdot P =$$

$$= 0.5 \cdot \frac{1}{\beta - \gamma} \cdot ((\beta - \gamma) \cdot (a + b \cdot P) - r - \tau - c).$$

Moreover, price index becomes

$$P = \int_0^N (p(i) + r(i) + \tau) \, di = N \cdot (p^*(r, P, N) + r + \tau) =$$

$$= 0.5 \cdot N \cdot ((\beta - \gamma) \cdot (a + b \cdot P) + r + \tau + c),$$

i.e.,

$$P(r, N) = \frac{N}{2 - (\beta - \gamma) \cdot b \cdot N} \cdot ((\beta - \gamma) \cdot a + r + \tau + c).$$

Hence, the second stage of the game responds to the retailer's choice r, N by demand

$$q^{\#}(r,N) = q^*\left(r, P\left(r,N\right), N\right) =$$

$$= 0.5 \cdot \frac{1}{\beta-\gamma} \cdot \left(\left(\beta-\gamma\right)\cdot\left(a+b\cdot P\left(r,N\right)\right)-r-\tau-c\right) =$$

$$= \frac{0.5}{\beta-\gamma} \cdot \left(\left(\beta-\gamma\right)\cdot\left(a+\frac{b\cdot N\cdot\left(\left(\beta-\gamma\right)\cdot a+r+\tau+c\right)}{2-\left(\beta-\gamma\right)\cdot b\cdot N}\right)-r-\tau-c\right) =$$

$$= \frac{1}{\beta-\gamma} \cdot \left(\frac{\left(\beta-\gamma\right)\cdot a}{2-\left(\beta-\gamma\right)\cdot b\cdot N}+\frac{\left(\beta-\gamma\right)\cdot b\cdot N-1}{2-\left(\beta-\gamma\right)\cdot b\cdot N}\cdot\left(r+\tau+c\right)\right) =$$

$$= \frac{\alpha-c-\tau-r}{2\cdot\left(\beta-\gamma\right)+\gamma\cdot N},$$

$$p^{\#}(r,N) = p^*\left(r, P\left(r,N\right), N\right) =$$

$$= 0.5\cdot\left(\left(\beta-\gamma\right)\cdot\left(a+b\cdot P\left(r,N\right)\right)-r-\tau+c\right) =$$

$$= 0.5\cdot\left(\left(\beta-\gamma\right)\cdot\left(a+b\cdot P\left(r,N\right)\right)-r-\tau-c\right)+c =$$

$$= \left(\beta-\gamma\right)\cdot q^{\#}(r,N)+c = \left(\beta-\gamma\right)\cdot\frac{\alpha-c-\tau-r}{2\cdot\left(\beta-\gamma\right)+\gamma\cdot N}+c.$$

2.3 Retailer

We plug the previous solutions and solve now the retailer's program

$$\pi_{\mathcal{R}}\left(r,N\right) = N\cdot\left(\left(r-c_{\mathcal{R}}\right)\cdot q^{\#}\left(r,N\right)-f_{\mathcal{R}}\right) =$$

$$= N\cdot\left(\left(r-c_{\mathcal{R}}\right)\cdot\frac{\alpha-\left(r+\tau+c\right)}{2\cdot\left(\beta-\gamma\right)+\gamma\cdot N}-\left(f_{\mathcal{R}}-f_E\right)\right) \to \max_{r,N},$$

subject to the manufacturer's no-losses constraint

$$\pi_{\mathcal{M}}\left(r,N\right) = \left(\beta-\gamma\right)\cdot\left(q^{\#}\left(r,N\right)\right)^2-f_{\mathcal{M}}-f_E \geq 0.$$

Differentiating profit, we get

$$\frac{\partial\pi_{\mathcal{R}}}{\partial r} = N\cdot\frac{\alpha-c+c_{\mathcal{R}}-\tau-2r}{2\cdot\left(\beta-\gamma\right)+\gamma\cdot N},$$

$$\frac{\partial\pi_{\mathcal{R}}}{\partial N} = 2\left(\beta-\gamma\right)\cdot\frac{\left(r-c_{\mathcal{R}}\right)\cdot\left(\alpha-c-\tau-r\right)}{\left(2\cdot\left(\beta-\gamma\right)+\gamma\cdot N\right)^2}-\left(f_{\mathcal{R}}-f_E\right).$$

For $N > 0$, if $f_{\mathcal{R}} > f_E$, then the only stationary point (r^0, N^0) of function $\pi_{\mathcal{R}}(r, N)$ (i.e., a point such that $\frac{\partial \pi_{\mathcal{R}}}{\partial r} = \frac{\partial \pi_{\mathcal{R}}}{\partial N} = 0$), is

$$r^0 = \frac{\alpha - c + c_{\mathcal{R}} - \tau}{2}\,, \qquad N^0 = \frac{\beta-\gamma}{\gamma} \cdot \left(\frac{\alpha - c - c_{\mathcal{R}} - \tau}{\sqrt{2\cdot(\beta-\gamma)\cdot(f_{\mathcal{R}} - f_E)}} - 2 \right).$$

Note that

$$r^0 - c_{\mathcal{R}} = \alpha - c - \tau - r^0 = \frac{\alpha - c - c_{\mathcal{R}} - \tau}{2} = \sqrt{\frac{f_{\mathcal{R}} - f_E}{2\cdot(\beta-\gamma)}} \cdot \left(2\cdot(\beta-\gamma) + \gamma\cdot N^0\right).$$

Further, to be sure in the profit concavity, we study the second derivatives

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial r^2} = -\frac{2\cdot N}{2\cdot(\beta-\gamma)+\gamma\cdot N}\,,$$

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial N^2} = -4\,(\beta-\gamma)\cdot\gamma\cdot\frac{(r-c_{\mathcal{R}})\cdot(\alpha-c-\tau-r)}{(2\cdot(\beta-\gamma)+\gamma\cdot N)^3}\,,$$

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial r \partial N} = 2\cdot(\beta-\gamma)\cdot\frac{\alpha - c + c_{\mathcal{R}} - \tau - 2r}{(2\cdot(\beta-\gamma)+\gamma\cdot N)^2} = -\frac{\beta-\gamma}{N^2}\cdot\frac{\partial^2 \pi_{\mathcal{R}}}{\partial r^2}\cdot\frac{\partial \pi_{\mathcal{R}}}{\partial r}$$

and

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial r^2}\left(r^0, N^0\right) = -\frac{2\cdot N^0}{2\,(\beta-\gamma)+\gamma\cdot N^0} =$$

$$= -2\cdot\frac{\left(\alpha - c - c_{\mathcal{R}} - \tau - 2\cdot\sqrt{2\cdot(\beta-\gamma)\cdot(f_{\mathcal{R}}-f_E)}\right)}{\gamma\cdot(\alpha - c - c_{\mathcal{R}} - \tau)} < 0,$$

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial N^2}\left(r^0, N^0\right) = -\frac{\gamma\cdot\left(\sqrt{2\cdot(f_{\mathcal{R}}-f_E)}\right)^3}{\sqrt{\beta-\gamma}\cdot(\alpha - c - c_{\mathcal{R}} - \tau)} < 0,$$

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial r \partial N}\left(r^0, N^0\right) = 0,$$

Hence the only stationary point (r^0, N^0) of function $\pi_{\mathcal{R}}(r, N)$ is the maximum point, not minimum or anything else.

The maximal value $\pi_{\mathcal{M}}(r^0, N^0)$ here is

$$\pi_{\mathcal{M}}\left(r^0, N^0\right) = (\beta-\gamma)\cdot\left(\frac{\alpha - c - \tau - r^0}{2\,(\beta-\gamma)+\gamma\cdot N^0}\right)^2 - f_{\mathcal{M}} - f_E =$$

$$= (\beta - \gamma) \cdot \left(\frac{\frac{\alpha - c - c_{\mathcal{R}} - \tau}{2}}{\sqrt{\frac{\beta - \gamma}{2 \cdot (f_{\mathcal{R}} - f_E)}} \cdot (\alpha - c - c_{\mathcal{R}} - \tau)} \right)^2 - f_{\mathcal{M}} - f_E =$$

$$= \frac{f_{\mathcal{R}} - f_E}{2} - f_{\mathcal{M}} - f_E = (f_{\mathcal{M}} + f_E) \cdot (F - 1),$$

where

$$F \equiv \frac{f_{\mathcal{R}} - f_E}{2 \cdot (f_{\mathcal{M}} + f_E)},$$

i.e.,

$$\pi_{\mathcal{M}} \left(r^0, N^0 \right) > 0 \Longleftrightarrow F > 1,$$

and we have proven which type of equilibria relate to which region of the parameters.

2.4 Equilibriun in Positive-Profit case $F > 1$

Denote constants

$$A \equiv \alpha - c - c_{\mathcal{R}},$$

$$B \equiv \sqrt{(\beta - \gamma) \cdot (f_{\mathcal{M}} + f_E)}.$$

If $F \equiv \frac{f_{\mathcal{R}} - f_E}{2 \cdot (f_{\mathcal{M}} + f_E)} > 1$ then

$$r_{pp} = r^0 = \frac{A - \tau}{2} + c_{\mathcal{R}}, \qquad N_{pp} = N^0 = \frac{\beta - \gamma}{\gamma} \cdot \left(\frac{A - \tau}{2 \cdot B \cdot \sqrt{F}} - 2 \right),$$

$$q_{pp} = \frac{\alpha - c - \tau - r_{pp}}{2 \cdot (\beta - \gamma) + \gamma \cdot N_{pp}} = \frac{B \cdot \sqrt{F}}{\beta - \gamma}, \qquad p_{pp} = B \cdot \sqrt{F} + c,$$

$$r^0 - c_{\mathcal{R}} = \alpha - c - \tau - r^0 = \frac{A - \tau}{2} = \frac{B \cdot \sqrt{F}}{\beta - \gamma} \cdot \left(2 \cdot (\beta - \gamma) + \gamma \cdot N^0 \right).$$

$$\pi_{\mathcal{R}} = N_{pp} \cdot ((r_{pp} - c_{\mathcal{R}}) \cdot q_{pp} - (f_{\mathcal{R}} - f_E)) =$$

$$= \frac{\beta - \gamma}{2\gamma} \cdot (f_{\mathcal{R}} - f_E) \cdot \left(\frac{A - \tau}{2 \cdot B \cdot \sqrt{F}} - 2 \right)^2 =$$

$$= \frac{B^2}{\gamma} \cdot F \cdot \left(\frac{A - \tau}{2 \cdot B \cdot \sqrt{F}} - 2 \right)^2 =$$

$$= \frac{\left(A - \tau - 4 \cdot B \cdot \sqrt{F} \right)^2}{4 \cdot \gamma}.$$

$$\pi_{\mathcal{M}} = \frac{f_{\mathcal{R}} - f_E}{2} - f_{\mathcal{M}} - f_E = (f_{\mathcal{M}} + f_E) \cdot (F - 1).$$

2.5 Equilibriun in Zero-Profit case $0 < F \leq 1$

Due to free entry (the manufacturer's zero-profit condition), we get

$$\pi_{\mathcal{M}}(r,N) = (\beta-\gamma)\cdot\left(q^{\#}(r,N)\right)^2 - f_{\mathcal{M}} - f_E = 0$$

$$N_{zp} = N_{zp}(r) = \frac{\beta-\gamma}{\gamma}\cdot\left(\frac{A+c_{\mathcal{R}}-(r+\tau)}{B}-2\right)$$

$$q_{zp}(r) = q^{\#}(r, N_{zp}(r)) = \frac{B}{\beta-\gamma},$$

$$\pi_{\mathcal{R}}(r) = \pi_{\mathcal{R}}(r, N_{zp}(r)) = N_{zp}(r)\cdot((r-c_{\mathcal{R}})\cdot q_{zp}(r) - (f_{\mathcal{R}}-f_E)) =$$

$$= \frac{\beta-\gamma}{\gamma}\cdot\left(\frac{A+c_{\mathcal{R}}-(r+\tau)}{B}-2\right)\cdot\left((r-c_{\mathcal{R}})\cdot\frac{B}{\beta-\gamma}-(f_{\mathcal{R}}-f_E)\right) =$$

$$= \frac{1}{\gamma}\cdot\left(\frac{A+c_{\mathcal{R}}-(r+\tau)}{B}-2\right)\cdot((r-c_{\mathcal{R}})\cdot B-(f_{\mathcal{R}}-f_E)\cdot(\beta-\gamma)) =$$

$$= -\frac{1}{\gamma}\cdot(r+\tau-A-c_{\mathcal{R}}+2\cdot B)\cdot(r-c_{\mathcal{R}}-2\cdot F\cdot B).$$

The First Order Condition is

$$\frac{d\pi_{\mathcal{R}}(r)}{dr} \equiv -\frac{2}{\gamma}\cdot\left(r-c_{\mathcal{R}}+\frac{\tau-A}{2}+B\cdot(1-F)\right) = 0,$$

it yields the solution

$$r_{zp} = \frac{A-\tau}{2}+c_{\mathcal{R}}+B\cdot(F-1),$$

$$N_{zp} = \frac{\beta-\gamma}{\gamma}\cdot\left(\frac{A+c_{\mathcal{R}}-(r_{zp}+\tau)}{B}-2\right) =$$

$$= \frac{\beta-\gamma}{\gamma}\cdot\left(\frac{A-\tau}{2\cdot B}-1-F\right),$$

$$q_{zp} = \frac{B}{\beta-\gamma},$$

$$p_{zp} = (\beta-\gamma)\cdot q_{zp}+c = B+c,$$

$$\pi_{\mathcal{R}} = N_{zp}\cdot((r_{zp}-c_{\mathcal{R}})\cdot q_{zp}-(f_{\mathcal{R}}-f_E)) =$$

$$= \frac{\beta-\gamma}{\gamma}\cdot\left(\frac{A-\tau}{2\cdot B}-1-F\right)\cdot\left(\left(\frac{A-\tau}{2}+B\cdot(F-1)\right)\cdot\frac{B}{\beta-\gamma}-(f_{\mathcal{R}}-f_E)\right) =$$

$$= \frac{1}{\gamma}\cdot\left(\frac{A-\tau}{2\cdot B}-1-F\right)\cdot\left(\left(\frac{A-\tau}{2}+B\cdot(F-1)\right)\cdot B-2\cdot B^2\cdot F\right) =$$

$$= \frac{1}{\gamma} \cdot \left(\frac{A-\tau}{2} - B \cdot (1+F) \right) \cdot \left(\frac{A-\tau}{2} + B \cdot (F-1) - 2 \cdot B \cdot F \right) =$$

$$= \frac{\left(A - \tau - 2 \cdot B \cdot (1+F)\right)^2}{2 \cdot \gamma}.$$

3 Entrance Fee imposed by retailer

To find which entrance fee chooses the retailer, we solve

$$\pi_{\mathcal{R}} = N \cdot (r - c_{\mathcal{R}})\, q\,(r, N) - N \cdot (f_{\mathcal{R}} - f_E) \;\rightarrow\; \max_{r,N,f_E}, \;\; \textbf{s.t.},$$

$$\pi_{\mathcal{M}} = (p\,(r, N) - c)\, q\,(r, N) - (f_{\mathcal{M}} + f_E) \geq 0.$$

Since

$$\frac{\partial \pi_{\mathcal{R}}}{\partial f_E} = N > 0,$$

only zero-profit "ZP" case can result from optimiziation. Again we denote

$$B \equiv B\,(f_E) = \sqrt{(\beta - \gamma) \cdot (f_{\mathcal{M}} + f_E)}, \qquad F = F\,(f_E) = \frac{f_{\mathcal{R}} - f_E}{2 \cdot (f_{\mathcal{M}} + f_E)}$$

and find

$$\frac{dB}{df_E} = \frac{\sqrt{\beta - \gamma}}{2 \cdot \sqrt{f_{\mathcal{M}} + f_E}} = \frac{\beta - \gamma}{2 \cdot B}\,,$$

$$\frac{dF}{df_E} = \frac{-\,(f_{\mathcal{M}} + f_E) - (f_{\mathcal{R}} - f_E)}{2 \cdot (f_{\mathcal{M}} + f_E)^2} = -\frac{1 + 2 \cdot F}{2 \cdot (f_{\mathcal{M}} + f_E)} = -\,(\beta - \gamma) \cdot \frac{1 + 2 \cdot F}{2 \cdot B^2}\,.$$

Further, we express the variables in these terms:

$$N = N\,(r, f_E) = \frac{\beta - \gamma}{\gamma} \cdot \left(\frac{A + c_{\mathcal{R}} - (r + \tau)}{B} - 2 \right),$$

$$q = q\,(r, f_E) = \frac{B}{\beta - \gamma}\,,$$

and express profit also:

$$\pi_{\mathcal{R}} = \pi_{\mathcal{R}}\,(r, f_E) = N \cdot ((r - c_{\mathcal{R}}) \cdot q - (f_{\mathcal{R}} - f_E)) =$$

$$= -\frac{1}{\gamma} \cdot (r - c_{\mathcal{R}} + \tau - A + 2 \cdot B) \cdot (r - c_{\mathcal{R}} - 2 \cdot F \cdot B)\,.$$

$$\frac{\partial \pi_{\mathcal{R}}}{\partial r} = -\frac{2}{\gamma} \cdot \left(r - c_{\mathcal{R}} - \left(\frac{A-\tau}{2} + B \cdot (F-1) \right) \right).$$

Further, to maximize profit, we get derivatives

$$\frac{\partial \pi_{\mathcal{R}}}{\partial f_E} = -\frac{2}{\gamma} \times$$

$$\times \left(\frac{dB}{df_E} \cdot (r - c_{\mathcal{R}} - 2 \cdot F \cdot B) - (r - c_{\mathcal{R}} + \tau - A + 2 \cdot B) \cdot \left(\frac{dB}{df_E} \cdot F + \frac{dF}{df_E} \cdot B \right) \right) =$$

$$= -\frac{2}{\gamma} \cdot \frac{\beta - \gamma}{2 \cdot B} \times$$

$$\times \left((r - c_{\mathcal{R}} - 2 \cdot F \cdot B) + (r - c_{\mathcal{R}} + \tau - A + 2 \cdot B) \cdot (1 + F) \right) =$$

$$= -\frac{2}{\gamma} \cdot \frac{\beta - \gamma}{2 \cdot B} \cdot \left((r - c_{\mathcal{R}}) \cdot (2 + F) + (\tau - A) \cdot (1 + F) + 2 \cdot B \right).$$

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial r^2} = -\frac{2}{\gamma}$$

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial r \partial f_E} = -\frac{2}{\gamma} \cdot \left(\frac{dB}{df_E} \cdot (1 - F) - \frac{dF}{df_E} \cdot B \right) =$$

$$= -\frac{2}{\gamma} \cdot \left(\frac{\beta - \gamma}{2 \cdot B} \cdot (1 - F) + (\beta - \gamma) \cdot \frac{1 + 2 \cdot F}{2 \cdot B} \right) =$$

$$= -\frac{2}{\gamma} \cdot \frac{\beta - \gamma}{2 \cdot B} \cdot (2 + F)$$

$$\frac{\partial^2 \pi_{\mathcal{R}}}{\partial \left(f_E \right)^2} = -\frac{\beta - \gamma}{2 \cdot \gamma} \times$$

$$\times \frac{(r - c_{\mathcal{R}} + \tau - A) \cdot \frac{dF}{df_E} \cdot B - \left((r - c_{\mathcal{R}}) \cdot (2 + F) + (\tau - A) \cdot (1 + F) \right) \cdot \frac{dB}{df_E}}{B^2} =$$

$$= \frac{(\beta - \gamma)^2}{4 \cdot \gamma \cdot B^3} \cdot \left(3 \cdot (r - c_{\mathcal{R}}) \cdot (1 + F) + (\tau - A) \cdot (2 + 3 \cdot F) \right).$$

Now a stationary point (r, f_E) we can find from the system

$$\frac{\partial \pi_{\mathcal{R}}}{\partial r} = -\frac{2}{\gamma} \cdot \left(r - c_{\mathcal{R}} - \left(\frac{A-\tau}{2} + B \cdot (F-1) \right) \right) = 0$$

$$\frac{\partial \pi_{\mathcal{R}}}{\partial f_E} = -\frac{2}{\gamma} \cdot \frac{\beta - \gamma}{2 \cdot B} \cdot \left((r - c_{\mathcal{R}}) \cdot (2 + F) + (\tau - A) \cdot (1 + F) + 2 \cdot B \right) = 0$$

i.e.,

$$r - c_{\mathcal{R}} = \frac{A - \tau}{2} + B \cdot (F - 1)$$

$$\left(\frac{A - \tau}{2} + B \cdot (F - 1) \right) \cdot (2 + F) + (\tau - A) \cdot (1 + F) + 2 \cdot B = 0$$

i.e.,

$$r - c_{\mathcal{R}} = \frac{A - \tau}{2} + B \cdot (F - 1)$$

$$\left(-\frac{A - \tau}{2} + B \cdot (F + 1) \right) \cdot F = 0$$

i.e.,

$$r - c_{\mathcal{R}} = \frac{A - \tau}{2} + B \cdot (F - 1)$$

$$\frac{\gamma}{\beta - \gamma} \cdot N \cdot F = 0$$

since

$$\frac{\partial \pi_{\mathcal{R}}}{\partial r} = 0 \Longleftrightarrow N = \frac{\beta - \gamma}{\gamma} \cdot \frac{\frac{A - \tau}{2} - B \cdot (F + 1)}{B} .$$

Therefore, if $N > 0$ then $F = 0$, i.e., the profit-maximizing solution is

$$f_{\mathcal{R}} = f_E.$$

This means that the retailer completely covers her fixed cost by the entrance fee from the supplier.

Further, plugging this solution we get

$$r = \frac{A - \tau}{2} + c_{\mathcal{R}} - B$$

$$N = \frac{\beta - \gamma}{\gamma} \cdot \left(\frac{A - \tau}{2 \cdot B} - 1 \right)$$

3.1 Summary of two-part-tariff and other equilibria

To put together the findings, recall notation

$$A \equiv \alpha - c - c_{\mathcal{R}}$$

$$B = \sqrt{(\beta - \gamma) \cdot (f_{\mathcal{M}} + f_E)}$$

$$F = \frac{f_{\mathcal{R}} - f_E}{2 \cdot (f_{\mathcal{M}} + f_E)}.$$

Then, we have derived

	$PP: \quad F>1$	$ZP: \quad 1 \geq F > 0$	$EF: \quad F = 0 = f_{\mathcal{R}} - f_E$
q	$\frac{B \cdot \sqrt{F}}{\beta-\gamma}$	$\frac{B}{\beta-\gamma}$	$\frac{B}{\beta-\gamma}$
p	$B \cdot \sqrt{F} + c$	$B + c$	$B + c$
r	$\frac{A-\tau}{2} + c_{\mathcal{R}}$	$\frac{A-\tau}{2} + c_{\mathcal{R}} - (1-F) \cdot B$	$\frac{A-\tau}{2} + c_{\mathcal{R}} - B$
N	$\frac{\beta-\gamma}{\gamma \cdot B} \cdot \left(\frac{A-\tau}{2 \cdot \sqrt{F}} - 2 \cdot B\right)$	$\frac{\beta-\gamma}{\gamma \cdot B} \cdot \left(\frac{A-\tau}{2} - (1+F) \cdot B\right)$	$\frac{\beta-\gamma}{\gamma \cdot B} \cdot \left(\frac{A-\tau}{2} - B\right)$
$\pi_{\mathcal{R}}$	$\frac{B^2}{\gamma} \cdot \left(\frac{A-\tau}{2 \cdot B} - 2 \cdot \sqrt{F}\right)^2$	$\frac{1}{\gamma} \cdot \left(\frac{A-\tau}{2} - B \cdot (1+F)\right)^2$	$\frac{1}{\gamma} \cdot \left(\frac{A-\tau}{2} - B\right)^2$
$\pi_{\mathcal{M}}$	$(f_{\mathcal{M}} + f_E) \cdot (F-1) > 0$	0	0

4 Welfare

4.1 Welfare expression under any (τ, f_E), before optimizing these

Plugging the usual equilibrium, welfare can be expressed as

$$W = N \cdot \left(q \cdot \left(A - \frac{1}{2} \cdot (\beta - \gamma + \gamma \cdot N) \cdot q\right) - (f_{\mathcal{M}} + f_{\mathcal{R}})\right) =$$

$$= N \cdot \left(q \cdot \left(A - \frac{1}{2} \cdot (\beta - \gamma + \gamma \cdot N) \cdot q\right) - (f_{\mathcal{M}} + f_E) \cdot \left(1 + \frac{f_{\mathcal{R}} - f_E}{f_{\mathcal{M}} + f_E}\right)\right) =$$

$$= N \cdot \left(q \cdot \left(A - \frac{1}{2} \cdot (\beta - \gamma + \gamma \cdot N) \cdot q\right) - \frac{B^2}{\beta-\gamma} \cdot (1 + 2 \cdot F)\right).$$

$$W_{pp} = N_{pp} \cdot \left(q_{pp} \cdot \left(A - \frac{1}{2} \cdot (\beta - \gamma + \gamma \cdot N_{pp}) \cdot q_{pp}\right) - \frac{B^2}{\beta-\gamma} \cdot (1 + 2 \cdot F)\right) =$$

$$W_{pp} = \frac{\beta-\gamma}{\gamma \cdot B} \cdot \left(\frac{A-\tau}{2 \cdot \sqrt{F}} - 2 \cdot B\right) \cdot \left(\frac{B \cdot \sqrt{F}}{\beta-\gamma} \cdot \left(A - \frac{1}{2} \cdot \left(\frac{A-\tau}{2 \cdot \sqrt{F}} - B\right) \cdot \sqrt{F}\right) - \frac{B^2}{\beta-\gamma} \cdot (1 + 2 \cdot F)\right) =$$

$$= \frac{1}{\gamma \cdot B} \cdot \left(\frac{A-\tau}{2 \cdot \sqrt{F}} - 2 \cdot B\right) \cdot \left(B \cdot \sqrt{F} \cdot \left(A - \frac{1}{2} \cdot \left(\frac{A-\tau}{2} - B \cdot \sqrt{F}\right)\right) - B^2 \cdot (1 + 2 \cdot F)\right) =$$

$$= \frac{1}{\gamma} \cdot \left(\frac{A-\tau}{2} - 2 \cdot B \cdot \sqrt{F}\right) \cdot \left(-\frac{1}{2} \cdot \left(\frac{A-\tau}{2} - B \cdot \sqrt{F}\right) + A - \frac{B \cdot (1 + 2 \cdot F)}{\sqrt{F}}\right) =$$

$$= \frac{1}{2 \cdot \gamma} \cdot \left(\frac{A-\tau}{2} - 2 \cdot B \cdot \sqrt{F}\right) \cdot \left(-\left(\frac{A-\tau}{2} - B \cdot \sqrt{F}\right) + 2 \cdot A - 2 \cdot \frac{B \cdot (1 + 2 \cdot F)}{\sqrt{F}}\right) =$$

$$= -\frac{1}{8 \cdot \gamma} \cdot \left(\tau - A + 4 \cdot B \cdot \sqrt{F}\right) \cdot \left(\tau + 2 \cdot B \cdot \sqrt{F} + 3 \cdot A - 4 \cdot \frac{B \cdot (1 + 2 \cdot F)}{\sqrt{F}}\right) =$$

$$= -\frac{1}{8 \cdot \gamma} \cdot \left(\tau - A + 4 \cdot B \cdot \sqrt{F}\right) \cdot \left(\tau + \frac{2 \cdot B \cdot F + 3 \cdot A \cdot \sqrt{F} - 4 \cdot B \cdot (1 + 2 \cdot F)}{\sqrt{F}}\right) =$$

$$= -\frac{1}{8 \cdot \gamma} \cdot \left(\tau - A + 4 \cdot B \cdot \sqrt{F}\right) \cdot \left(\tau + 3 \cdot A - 2 \cdot \left(\frac{2}{\sqrt{F}} + 3 \cdot \sqrt{F}\right) \cdot B\right).$$

4.2 Welfare maximization in (τ)

Now we can optimize welfare in taxes:

$$\frac{dW_{pp}}{d\tau} = -\frac{1}{4 \cdot \gamma} \cdot \left(\tau + A - \left(\frac{2}{\sqrt{F}} + \sqrt{F}\right) \cdot B\right).$$

$$\tau_{pp}^{W} = B \cdot \left(\frac{2}{\sqrt{F}} + \sqrt{F}\right) - A.$$

$$W_{pp}^{W} = -\frac{1}{8 \cdot \gamma} \cdot \left(\tau_{pp}^{W} - A + 4 \cdot B \cdot \sqrt{F}\right) \cdot \left(\tau_{pp}^{W} + 3 \cdot A - 2 \cdot B \cdot \left(\frac{2}{\sqrt{F}} + 3 \cdot \sqrt{F}\right)\right) =$$

$$= -\frac{1}{8 \cdot \gamma} \cdot \left(B \cdot \left(\frac{2}{\sqrt{F}} + \sqrt{F}\right) - A - A + 4 \cdot B \cdot \sqrt{F}\right) \times$$

$$\times \left(B \cdot \left(\frac{2}{\sqrt{F}} + \sqrt{F}\right) - A + 3 \cdot A - 2 \cdot B \cdot \left(\frac{2}{\sqrt{F}} + 3 \cdot \sqrt{F}\right)\right) =$$

$$= -\frac{1}{8 \cdot \gamma} \cdot \left(B \cdot \left(\frac{2}{\sqrt{F}} + 5 \cdot \sqrt{F}\right) - 2 \cdot A\right) \cdot \left(2 \cdot A - B \cdot \left(\frac{2}{\sqrt{F}} + 5 \cdot \sqrt{F}\right)\right) =$$

$$= \frac{1}{8 \cdot \gamma} \cdot \left(2 \cdot A - B \cdot \left(\frac{2}{\sqrt{F}} + 5 \cdot \sqrt{F}\right)\right)^2.$$

$$W_{zp} = N_{zp} \cdot \left(q_{zp} \cdot \left(A - \frac{1}{2} \cdot (\beta - \gamma + \gamma \cdot N_{zp}) \cdot q_{zp}\right) - \frac{B^2}{\beta - \gamma} \cdot (1 + 2 \cdot F)\right) =$$

$$= \frac{1}{\gamma} \cdot \left(\frac{A - \tau}{2} - (1 + F) \cdot B\right) \cdot \left(A - \frac{1}{2} \cdot \left(\frac{A - \tau}{2} - F \cdot B\right) - (1 + 2 \cdot F) \cdot B\right) =$$

$$= -\frac{1}{2 \cdot \gamma} \cdot \left(\frac{A - \tau}{2} - (1 + F) \cdot B\right) \cdot \left(\frac{A - \tau}{2} - 2 \cdot A + (2 + 3 \cdot F) \cdot B\right) =$$

$$= -\frac{1}{8 \cdot \gamma} \cdot (\tau - A + 2 \cdot (1 + F) \cdot B) \cdot (\tau + 3 \cdot A - 2 \cdot (2 + 3 \cdot F) \cdot B).$$

$$\frac{dW_{zp}}{d\tau} = -\frac{1}{4 \cdot \gamma} \cdot (\tau + A - (1 + 2 \cdot F) \cdot B).$$

$$\tau_{zp}^{W} = (1 + 2 \cdot F) \cdot B - A.$$

$$W_{zp}^{W} = -\frac{1}{8\cdot\gamma}\cdot\left(\tau_{zp}^{W} - A + 2\cdot(1+F)\cdot B\right)\cdot\left(\tau_{zp}^{W} + 3\cdot A - 2\cdot(2+3\cdot F)\cdot B\right) =$$

$$= \frac{1}{8\cdot\gamma}\cdot(2\cdot A - (3+4\cdot F)\cdot B)^2$$

Hence, under best tax/subsidy we get equilibria

	W
$PP: \quad F>1$	$-\frac{1}{8\cdot\gamma}\cdot\left(\tau - A + 4\cdot B\cdot\sqrt{F}\right)\cdot\left(\tau + 3\cdot A - 2\cdot\left(\frac{2}{\sqrt{F}} + 3\cdot\sqrt{F}\right)\cdot B\right)$
$ZP: \quad 1\geq F>0$	$-\frac{1}{8\cdot\gamma}\cdot(\tau - A + 2\cdot(1+F)\cdot B)\cdot(\tau + 3\cdot A - 2\cdot(2+3\cdot F)\cdot B)$
$EF: \quad F=0$	$-\frac{1}{8\cdot\gamma}\cdot(\tau - A + 2\cdot B)\cdot(\tau + 3\cdot A - 4\cdot B)$

and

	τ^{W}	$W\left(\tau^{W}\right)$
$PP: \quad F>1$	$B\cdot\left(\frac{2}{\sqrt{F}} + \sqrt{F}\right) - A$	$\frac{1}{8\cdot\gamma}\cdot\left(2\cdot A - B\cdot\left(\frac{2}{\sqrt{F}} + 5\cdot\sqrt{F}\right)\right)^2$
$ZP: \quad 1\geq F>0$	$(1+2\cdot F)\cdot B - A$	$\frac{1}{8\cdot\gamma}\cdot(2\cdot A - (3+4\cdot F)\cdot B)^2$
$EF: \quad F=0$	$B - A$	$\frac{1}{8\cdot\gamma}\cdot(2\cdot A - 3\cdot B)^2$

Moreover,

	$PP: \quad F>1$	$ZP: \quad 1\geq F>0$	$EF: \quad F=0=f_{\mathcal{R}}-f_{E}$
τ^{W}	$-A + \frac{2+F}{\sqrt{F}}\cdot B$	$-A + (1+2\cdot F)\cdot B$	$B - A$
r^{W}	$A + c_{\mathcal{R}} - \frac{2+5\cdot F}{2\cdot\sqrt{F}}\cdot B$	$A + c_{\mathcal{R}} - \frac{3}{2}\cdot B$	$A + c_{\mathcal{R}} - \frac{3}{2}\cdot B$
N^{W}	$\frac{\beta-\gamma}{\gamma\cdot\sqrt{F}}\cdot\left(\frac{A}{B} - \frac{2+5\cdot F}{2\cdot\sqrt{F}}\right)$	$\frac{\beta-\gamma}{\gamma}\cdot\left(\frac{A}{B} - \frac{3+4\cdot F}{2}\right)$	$\frac{\beta-\gamma}{\gamma}\cdot\left(\frac{A}{B} - \frac{3}{2}\right)$
$\pi_{\mathcal{R}}^{W}$	$2\cdot W\left(\tau_{pp}^{W}\right)$	$2\cdot W\left(\tau_{zp}^{W}\right)$	$2\cdot W\left(\tau_{EF}^{W}\right)$
$\pi_{\mathcal{M}}$	$(f_{\mathcal{M}} + f_{E})\cdot(F_{*} - 1) > 0$	0	0

Note that if

$$N_{pp}^{W} = \frac{\beta-\gamma}{\gamma\cdot\sqrt{F}}\cdot\left(\frac{A}{B} - \frac{2+5\cdot F}{2\cdot\sqrt{F}}\right) > 0,$$

i.e.,

$$B < \frac{2\cdot\sqrt{F}}{2+5\cdot F}\cdot A,$$

then

$$\tau_{pp}^{W} = -A + \frac{2+F}{\sqrt{F}} \cdot B < -A + \frac{2+F}{\sqrt{F}} \cdot \frac{2 \cdot \sqrt{F}}{2 + 5 \cdot F} \cdot A =$$

$$= -3 \cdot \frac{F - \frac{2}{3}}{2 + 5 \cdot F} \cdot A < 0.$$

Moreover, if

$$N_{zp}^{W} = \frac{\beta - \gamma}{\gamma} \cdot \left(\frac{A}{B} - \frac{3 + 4 \cdot F}{2} \right) > 0,$$

i.e.,

$$B < \frac{2}{3 + 4 \cdot F} \cdot A,$$

then

$$\tau_{zp}^{W} = -A + (1 + 2 \cdot F) \cdot B < -A + \frac{2 \cdot (1 + 2 \cdot F)}{3 + 4 \cdot F} \cdot A =$$

$$= \frac{-A}{3 + 4 \cdot F} < 0;$$

and, of course, if

$$N_{EF}^{W} = \frac{\beta - \gamma}{\gamma} \cdot \left(\frac{A}{B} - \frac{3}{2} \right) > 0,$$

i.e.,

$$B < \frac{2}{3} \cdot A,$$

then

$$\tau_{zp}^{W} = B - A < \frac{2}{3} \cdot A - A = -\frac{A}{3} < 0.$$

5 Socially optimal (q, N) compared with equilibria

Consider the direct welfare maximization by a benevolent government:

$$W = N \cdot \left(q \cdot \left(A - \frac{1}{2} \cdot (\beta - \gamma + \gamma \cdot N) \cdot q \right) - (f_{\mathcal{M}} + f_{\mathcal{R}}) \right) \rightarrow \max_{q,N}$$

$$\frac{\partial W}{\partial q} = N \cdot (A - (\beta - \gamma + \gamma \cdot N) \cdot q)$$

$$\frac{\partial W}{\partial N} = q \cdot \left(A - \frac{1}{2} \cdot (\beta - \gamma + \gamma \cdot N) \cdot q \right) - (f_{\mathcal{M}} + f_{\mathcal{R}}) - \frac{\gamma}{2} \cdot N \cdot q^2$$

$$\frac{\partial^2 W}{\partial q^2} = -N \cdot (\beta - \gamma + \gamma \cdot N)$$

$$\frac{\partial^2 W}{\partial N^2} = -\gamma \cdot q^2 < 0$$

$$\frac{\partial^2 W}{\partial q \partial N} = A - (\beta - \gamma + 2 \cdot \gamma \cdot N) \cdot q$$

Stationary point is the positive solution of the system

$$A - (\beta - \gamma + \gamma \cdot N) \cdot q = 0,$$

$$q \cdot \left(A - \frac{1}{2} \cdot (\beta - \gamma + \gamma \cdot N) \cdot q \right) - (f_{\mathcal{M}} + f_{\mathcal{R}}) - \frac{\gamma}{2} \cdot N \cdot q^2 = 0,$$

i.e.,

$$A - (\beta - \gamma + \gamma \cdot N) \cdot q = 0,$$

$$q^2 = 2 \cdot \frac{f_{\mathcal{M}} + f_{\mathcal{R}}}{\beta - \gamma} = 2 \cdot \frac{B^2 \cdot (1 + 2 \cdot F)}{(\beta - \gamma)^2},$$

i.e.,

$$q = \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot \frac{B}{\beta - \gamma},$$

$$N = \frac{\beta - \gamma}{\gamma \cdot B} \cdot \frac{A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B}{\sqrt{2 \cdot (1 + 2 \cdot F)}}.$$

Then, in stationary point,

$$\frac{\partial^2 W}{\partial q^2} = -\frac{(\beta - \gamma)^2}{\gamma} \cdot \frac{A \cdot \left(A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B \right)}{2 \cdot (1 + 2 \cdot F) \cdot B^2} < 0$$

(since $N > 0$)

$$\frac{\partial^2 W}{\partial N^2} = -2 \cdot \gamma \cdot (1 + 2 \cdot F) \cdot \frac{B^2}{(\beta - \gamma)^2} < 0$$

$$\frac{\partial^2 W}{\partial q \partial N} = -A + \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B$$

$$\frac{\partial^2 W}{\partial q^2} \cdot \frac{\partial^2 W}{\partial N^2} - \left(\frac{\partial^2 W}{\partial q \partial N} \right)^2 =$$

$$= A \cdot \left(A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B \right) - \left(A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B \right)^2 =$$

$$= \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B \cdot \left(A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B \right) > 0.$$

Therefore, the stationary point

$$q^{sopt} = \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot \frac{B}{\beta - \gamma},$$

$$N^{sopt} = \frac{\beta - \gamma}{\gamma \cdot B} \cdot \frac{A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B}{\sqrt{2 \cdot (1 + 2 \cdot F)}}$$

is a maximum point. Further,

$$W\left(q^{sopt}, N^{sopt}\right) = N^{sopt} \cdot \left(q^{sopt} \cdot \left(A - \frac{1}{2} \cdot \left(\beta - \gamma + \gamma \cdot N^{sopt}\right) \cdot q^{sopt} \right) - \left(f_{\mathcal{M}} + f_{\mathcal{R}}\right) \right) =$$

$$= N^{sopt} \cdot \left(q^{sopt} \cdot \left(A - \frac{1}{2} \cdot \left(\beta - \gamma + \gamma \cdot N^{sopt}\right) \cdot q^{sopt} \right) - \frac{B^2}{\beta - \gamma} \cdot (1 + 2 \cdot F) \right) =$$

$$= \frac{1}{2 \cdot \gamma} \cdot \frac{A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B}{\sqrt{2 \cdot (1 + 2 \cdot F)}} \cdot \left(\sqrt{2 \cdot (1 + 2 \cdot F)} \cdot A - 2 \cdot (1 + 2 \cdot F) \cdot B \right) =$$

$$= \frac{1}{2 \cdot \gamma} \cdot \left(A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B \right)^2.$$

The table:

	W
$PP: \quad F > 1$	$\frac{1}{8 \cdot \gamma} \cdot \left(2 \cdot A - B \cdot \left(\frac{2}{\sqrt{F}} + 5 \cdot \sqrt{F} \right) \right)^2$
$ZP: \quad 1 \geq F > 0$	$\frac{1}{8 \cdot \gamma} \cdot (2 \cdot A - (3 + 4 \cdot F) \cdot B)^2$
$EF: \quad F = 0$	$\frac{1}{8 \cdot \gamma} \cdot (2 \cdot A - 3 \cdot B)^2$
Social optimality	$\frac{1}{8 \cdot \gamma} \cdot \left(2 \cdot A - 2 \cdot \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B \right)^2$

Note that in any case, $W^{sopt} > W\left(\tau_{zp}^W\right)$ and $W^{sopt} > W\left(\tau_{zp}^W\right)$. Indeed, since $N\left(\tau_{pp}^W\right) > 0$ and $N^{sopt} > 0$,

$$W^{sopt} > W\left(\tau_{pp}^W\right) \Longleftrightarrow \frac{2}{\sqrt{F}} + 5 \cdot \sqrt{F} > 2 \cdot \sqrt{2 \cdot (1 + 2 \cdot F)},$$

i.e.,

$$\frac{4}{F} + 25 \cdot F + 20 > 8 \cdot (1 + 2 \cdot F).$$

But

$$9 \cdot F^2 + 12 \cdot F + 4 = (3 \cdot F + 2)^2 > 0$$

since $F > 0$.

Analogously, since $N\left(\tau_{zp}^W\right) > 0$ and $N^{sopt} > 0$,

$$W^{sopt} > W\left(\tau_{zp}^W\right) \Longleftrightarrow 3 + 4 \cdot F > 2 \cdot \sqrt{2 \cdot (1 + 2 \cdot F)},$$

i.e.,

$$9 + 24 \cdot F + 16 \cdot F^2 > 8 \cdot (1 + 2 \cdot F).$$

But

$$1 + 8 \cdot F + 16 \cdot F^2 = (4 \cdot F + 1)^2 > 0.$$

since $F > 0$.

Further,

$$q^{sopt} - q_{pp}^W = \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot \frac{B}{\beta - \gamma} - \frac{B \cdot \sqrt{F}}{\beta - \gamma} = \left(\sqrt{2 \cdot (1 + 2 \cdot F)} - \sqrt{F}\right) \cdot \frac{B}{\beta - \gamma} =$$

$$= \frac{\left(\sqrt{2 \cdot (1 + 2 \cdot F)} - \sqrt{F}\right) \cdot \left(\sqrt{2 \cdot (1 + 2 \cdot F)} + \sqrt{F}\right)}{\sqrt{2 \cdot (1 + 2 \cdot F)} + \sqrt{F}} \cdot \frac{B}{\beta - \gamma} =$$

$$= \frac{2 \cdot (1 + 2 \cdot F) - F}{\sqrt{2 \cdot (1 + 2 \cdot F)} + \sqrt{F}} \cdot \frac{B}{\beta - \gamma} = \frac{2 + 3 \cdot F}{\sqrt{2 \cdot (1 + 2 \cdot F)} + \sqrt{F}} \cdot \frac{B}{\beta - \gamma} > 0,$$

$$q^{sopt} - q_{zp}^W = \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot \frac{B}{\beta - \gamma} - \frac{B}{\beta - \gamma} = \left(\sqrt{2 \cdot (1 + 2 \cdot F)} - 1\right) \cdot \frac{B}{\beta - \gamma} =$$

$$= \frac{\left(\sqrt{2 \cdot (1 + 2 \cdot F)} - 1\right) \cdot \left(\sqrt{2 \cdot (1 + 2 \cdot F)} + 1\right)}{\sqrt{2 \cdot (1 + 2 \cdot F)} + 1} \cdot \frac{B}{\beta - \gamma} =$$

$$= \frac{2 \cdot (1 + 2 \cdot F) - 1}{\sqrt{2 \cdot (1 + 2 \cdot F)} + 1} \cdot \frac{B}{\beta - \gamma} = \frac{2 + 3 \cdot F}{\sqrt{2 \cdot (1 + 2 \cdot F)} + 1} \cdot \frac{B}{\beta - \gamma} > 0,$$

$$N^{sopt} - N_{pp}^W = \frac{\beta - \gamma}{\gamma \cdot B} \cdot \frac{A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - \frac{\beta - \gamma}{\gamma \cdot \sqrt{F}} \cdot \left(\frac{A}{B} - \frac{2 + 5 \cdot F}{2 \cdot \sqrt{F}}\right) =$$

$$= \left(\left(\frac{A}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - B\right) - \left(\frac{1}{\sqrt{F}} \cdot A - \frac{2 + 5 \cdot F}{2 \cdot F} \cdot B\right)\right) \cdot \frac{\beta - \gamma}{\gamma \cdot B} =$$

$$= \left(\left(\frac{1}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - \frac{1}{\sqrt{F}}\right) \cdot A + \frac{2 + 3 \cdot F}{2 \cdot F} \cdot B\right) \cdot \frac{\beta - \gamma}{\gamma \cdot B} =$$

$$= \left(\left(\frac{1}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - \frac{1}{\sqrt{F}} \right) \cdot A + \frac{2 + 3 \cdot F}{2 \cdot F} \cdot B \right) \cdot \frac{\beta - \gamma}{\gamma \cdot B} ,$$

$$N^{sopt} - N_{zp}^{W} = \frac{\beta - \gamma}{\gamma \cdot B} \cdot \frac{A - \sqrt{2 \cdot (1 + 2 \cdot F)} \cdot B}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - \frac{\beta - \gamma}{\gamma} \cdot \left(\frac{A}{B} - \frac{3 + 4 \cdot F}{2} \right) =$$

$$= \left(\left(\frac{A}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - B - A + \frac{3 + 4 \cdot F}{2} \cdot B \right) \cdot \frac{\beta - \gamma}{\gamma \cdot B} \right) =$$

$$= \left(\left(\left(\frac{1}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - 1 \right) \cdot A + \frac{1 + 4 \cdot F}{2} \cdot B \right) \cdot \frac{\beta - \gamma}{\gamma \cdot B} \right) .$$

Therefore,

$$q^{sopt} > q_{pp}^{W}, \qquad q^{sopt} > q_{zp}^{W},$$

$$N^{sopt} < N_{pp}^{W} \Longleftrightarrow \left(\frac{1}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - \frac{1}{\sqrt{F}} \right) \cdot A + \frac{2 + 3 \cdot F}{2 \cdot F} \cdot B < 0,$$

$$N^{sopt} < N_{zp}^{W} \Longleftrightarrow \left(\frac{1}{\sqrt{2 \cdot (1 + 2 \cdot F)}} - 1 \right) \cdot A + \frac{1 + 4 \cdot F}{2} \cdot B < 0.$$

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