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Term Structure of Russian Credit Rates and Arbitrage Theory

Victor Chetverikov[a], Vadim Ajevsky[b] and Mariya Bondareva[a]

[a]National Research University Higher School of Economics, Moscow, Russia,
vchetverikov@hse.ru

[b]JSC VTB Bank, Moscow, Russia
ajevsky@msk.vtb.ru

Abstract

The credit market can be considered as a certain analogue of the zero coupon bond market, where credit granting is like bond purchasing, and receiving the credit is like the short sale of bonds. Of course, there is no perfect analogy because the operational procedures on these markets are rather different. However, it is assumed in this article that term structures of these rates of return are similar. For zero-coupon bonds in a time-discrete model there is a known affine temporal structure of yield to maturity that depends on the current short-term rate. In this article the one week interest rate was used as the yield to maturity for one-period bonds. An autoregressive scheme AR (1) for a one week rate of interest was taken as a Vasiček discrete model for the short rate of interest. Using a latent scalar parameter which exists in the model allows constructing adequate term structure of interest rates on the Russian market of interbank credits.

Keywords: Lending Rates, Yield to Maturity, Term Structure, Stochastic Discount Factor

Introduction

In the theory and practice of asset pricing, the main approach is based on the assumption that it is arbitrage-free, i.e., on the *arbitrage-free principle* (AFP) (Shiryaev, A.N. (1998); Follmer, H. & Schied, A. (2004); Safarian Mher & Kabanov Yuri. (2010)). It means that two portfolios with equal future random payments (*equivalent* portfolios) must have the same current price. Otherwise, arbitrage becomes possible (i.e., it is possible to earn a «free breakfast»), namely, the owner of the portfolio with a higher price can sell it and buy an equivalent portfolio with a lower price and thus obtain an income equal to the difference between the prices, because the expected future payments in both portfolios are the same. According to the AFP, the price of the financial portfolio is determined as the price of the equivalent portfolio with simplest assets whose prices can easily be calculated.

In the present paper, the authors followed Cochrane's approach (Cochrane, J. (2005)), whose main idea is that the determining factor of pricing (its kernel) is the *stochastic discount factor*. But, in contrast to Cochrane, the main attention was paid to the term structure and pricing of interest rates in the framework of the so-called Duffie-Kan affine models (Duffie, D. & Kan, R. (1996)) with discrete time. It is allowed to find temporary structure yield to maturity zero-coupon bonds if one uses a discrete Vasiček model for the short-term rate (Vasiček, O. (1977); Ajevsky, V.V. & Chetverikov, V.M. (2012)). In this structure, there is one free parameter, hereinafter referred to as latency. These issues are discussed in the first section of this paper.

In the second section there is a discussion about the relationship between the observable variables of this model and the latent variable characterizing the investor attitude to the risks due to variations in the stochastic discount factor.

Further the relations obtained for the yield to maturity of zero-coupon bonds are used for the interest rates on the Russian credit market. Obviously, the yield to maturity of one-period bonds in a discrete time model looks like the lending rate for the same period. It is assumed that interest rates for any period are subject to the same laws as the yield to maturity zero-coupon bonds for the same period. Strictly speaking, this is only a hypothesis about *the equivalence of temporary structures of credit rates and yield to maturity*. In reality, bonds and loans are not the same so as the loans cannot generally be sold before the deadline, unlike the bonds traded on the secondary market.

In the third section, it was shown how the model in question can be used to analyze the lending rates on the Moscow market of interbank credits (MMIBC, «MosPrime»). The data for these rates are presented in the Appendix. Five figures show that using a single latent variable as the "gauge parameter", we can achieve a

good agreement between the calculated yield of loans (deposits) and the observed rates. Note that only a single value of "gauge parameter" was used for five different credit durations at a time.

Remark. In the case of continuous models, the econometric estimation of interest rates was performed in 2003 on the basis of the MMIBC data by Anatoliev and Korepanov (Anatolyev, S.& Korepanov, S. (2003)). Bjork's monograph (Bjork, T. (2004)) contains a detailed survey of the last achievements in the field of continuous time models. A rather complete econometric investigation of statistical properties of GKO maturing yield was performed by Drobyshevskii (Drobyshevskii, S. (1999)).

Construction of a Pricing Model

Consider a pricing model on the zero-coupon market based on the following hypotheses and theorems.

Proposition 1. For any t, the logarithm of the price of one-period bond is given by the formula

$$\ln b_t^1 = -r_t, \quad \ln b_{t+1}^1 = -r_{t+1}, \tag{1}$$

where r_t is the so-called short-term rate whose dynamics is described by the following hypothesis.

Proposition 2. The short-term rate varies in discrete time by the formula

$$r_{t+1} = \varphi \cdot r_t + \theta \cdot (1-\varphi) + [w_0]^{1/2} \cdot \varepsilon_{t+1}, \tag{2}$$

where $w_0 \geq 0,$ and all ε_{t+1} are independent identically distributed variables for distinct t and $\varepsilon_{t+1} \in N(0,1)$. In financial mathematics, Eq. (2) is called the Vasiček discrete model (Vasiček, O. (1977)), but in the physical literature it is usually called the Ornstein-Uhlenbeck discrete model (Klyatskin, V.I. (1975); Pitovranov, S. E. & Chetverikov, V. M. (1978); Pitovranov, S. E. & Chetverikov, V. M. (1980)).

Proposition 3. The condition that the bond market is one-period arbitrage-free is satisfied if the prices of an n-period bond at time t are determined by the conditional mathematical expectation of the discounted price of an $n-1$-period bond at time $t=1$, i.e,

$$b_t^n = E_t\{m_{t+1} \cdot b_{t+1}^{n-1}\}, \quad b_t^0 = 1 \quad \forall t. \tag{3}$$

Here $E_t\{...\}$ means averaging over a measure related to the realization ε_{t+1}, and m_{t+1} is a stochastic discount factor depending on ε_{t+1}.

Proposition 4. The stochastic discount factor is determined by the short-term rate r_t, the random variable ε_{t+1}, and two constants δ and λ by the formula

$$-\ln m_{t+1} = \delta + \gamma \cdot r_t + \lambda \cdot [w_0]^{1/2} \varepsilon_{t+1} \quad . \tag{4}$$

Proposition 4 implies the following obvious corollary.

Corollary 1. The random variable m_{t+1} has a log-normal conditional distribution

$$\ln m_{t+1} \in N(-\delta - \gamma \cdot r_t, \quad \lambda^2 \cdot w_0), \tag{5}$$

and hence direct calculations easily prove that the logarithm of the mean is determined by the logarithm mean plus haft the logarithm variance:

$$\ln E_t\{m_{t+1}\} = E_t\{\ln m_{t+1}\} + \frac{1}{2} \cdot D\{\ln m_{t+1}\} = -\delta - \gamma \cdot r_t + \frac{\lambda^2}{2} \cdot w_0. \tag{6}$$

Corollary 2. Since Proposition 3 and Corollary 1 imply

$$\ln b_t^1 = \ln E_t\{m_{t+1} \cdot b_{t+1}^0\} = \ln E_t\{m_{t+1} \cdot 1\} = -\delta - \gamma \cdot r_t + \frac{\lambda^2}{2} \cdot w_0 ,$$

the following relation between the constants is required for the consistency with Proposition 2:

$$\delta = 0,5\lambda^2 \cdot w_0, \quad \gamma = 1 . \tag{7}$$

Corollary 3.

$$\ln b_t^2 = \ln E_t\{m_{t+1} \cdot b_{t+1}^1\} = \ln E_t\{\exp(-\delta - \gamma \cdot r_t - \lambda \cdot [w_0]^{1/2} \cdot \varepsilon_{t+1} - r_{t+1})\} =$$
$$= 0,5 \cdot (\lambda + 1)^2 \cdot w_0 - (\delta + \theta \cdot (1 - \varphi)) - (\gamma + \varphi) \cdot r_t$$

or with Corollary 2 taken into account,

$$\ln b_t^2 = -\theta \cdot (1 - \varphi) + (\lambda + \frac{1}{2}) \cdot w_0 - (1 + \varphi) \cdot r_t \tag{8}$$

Theorem. If the assumptions of Propositions 1-4 are satisfied, then the price of an n -period bond b_t^n depends on the time t only through the value of the short-term rate r_t :

$$-\ln b_t^n = A_n + B_n \cdot r_t , \tag{9}$$

and the coefficients A_n, B_n are independent of time.

These coefficients satisfy the system of recursive equations

$$\begin{aligned} &A_{n+1} = A_n + [\theta \cdot (1 - \varphi) - w_0 \cdot \lambda] \cdot B_n - 0,5 w_0 \cdot B_n{}^2, \\ &B_{n+1} = 1 + \varphi \cdot B_n, \\ &A_0 = A_1 = 0, \quad B_0 = 0, \quad B_1 = 1. \end{aligned} \tag{10}$$

This theorem is proved by the method of mathematical induction on n . By formula (8), assertions (9)-(10) hold for $n = 0$ and $n = 1$. It is assumed that they hold for n and prove them for $n + 1$. By Proposition 3 stating that the bond pricing is arbitrage-free, one has

$$\ln b_t^{n+1} = \ln E_t\{m_{t+1} \cdot b_{t+1}^n\} =$$
$$= \ln E_t\{\exp(-0,5\lambda^2 \cdot w_0 - r_t - \lambda \cdot [w_0]^{1/2} \cdot \varepsilon_{t+1} - A_n - B_n \cdot r_{t+1})\} =$$
$$= -[A_n + B_n \cdot (\theta \cdot (1 - \varphi) - \lambda \cdot w_0) - 0,5 w_0 \cdot B_n{}^2] - [1 + \varphi \cdot B_n] \cdot r_t = -A_{n+1} - B_{n+1} \cdot r_t .$$

Comparison of the last two rows shows that recursive relations (10) are satisfied. □

For a short-term rate r_t whose dynamics is modeled by Eq. (2), the following relations for the conditional means and variances of the variable r_{t+n} are correct:

$$E_t\{r_{t+n}\} = \varphi^n \cdot r_t + \theta \cdot (1 - \varphi^n), \tag{11}$$

$$D_t\{r_{t+n}\} = w_0 \cdot BB_n , \tag{12}$$

where the coefficients BB_n are determined by the relations

$$BB_n = \sum_{k=0}^{n-1} \varphi^{2k} = \frac{1-\varphi^{2n}}{1-\varphi^2}. \qquad (13)$$

Formulas (11)-(13) are proved by the method of mathematical induction.

In relations (9), (10) determining the arbitrage-free prices of the bonds, all variables except the constant λ are determined by Eq. (2) for the short-term rate, which is quite natural. The constant λ first appeared in the stochastic discounting coefficient (4). With (7) taken into account, the logarithm of this coefficient is determined by the formula

$$-\ln m_{t+1} = r_t + \frac{\lambda^2}{2} \cdot w_0 + \lambda \cdot [w_0]^{1/2} \varepsilon_{t+1} \quad . \qquad (14)$$

Formula (14) means that the constant λ together with the conditional variance $D_t\{r_{t+1}\} = w_0$ and the random variable ε_{t+1} determine the stochastic discounting coefficient m_{t+1} deviation from the «natural» quantity $E_t\{m_{t+1}\} = \exp(-r_t)$ depending only on the short-term rate r_t.

It follows from formulas (2) and (14) that $COV_t\{\ln m_{t+1}, r_{t+1}\} = -\lambda \cdot w_0$, hence

$$corr_t\{\ln m_{t+1}, r_{t+1}\} = -\lambda \cdot |\lambda|^{-1}. \qquad (15)$$

In other words, the modulus of the logarithm of the correlation of stochastic discount factor and the short rate is equal to one, and the sign is determined by the sign of the constant λ

Observable Variables and a Latent Parameter λ.

To test the hypothesis of the equivalence of temporary structures stated in the introduction, let us consider the temporal structure of zero-coupon bonds yield to maturity, following this model of pricing bonds.

It follows from the meaning of continuous rates used in the present paper that the maturing yield of an n-period bond at time t is given by the formula

$$y_t^n = -\frac{1}{n} \cdot \ln b_t^n \quad . \qquad (16)$$

In the proposed model of arbitrage-free pricing (9), the yield is determined by the formula

$$Y_t^n = -\frac{A_n + B_n \cdot r_t}{n}, \qquad . \qquad (17)$$

where A_n, B_n are determined by the recursive relations (10).

In formulas (10), all values except constants λ have a very definite economic meaning as a defined econometric model of the short-term rate (2).

In the general initial model, the parameter λ first appears in (4) and determines the value of the linear influence of the random factor ε_{t+1} on $\ln m_{t+1}$. For $\lambda = 0$, the stochastic discounting factor (4) has the form $m_{t+1} = \exp(-r_t)$; in this case, the discounting at time t of the future price at time $t+1$ depends only on the current short-term rate r_t without taking account of its possible variations under the action of the random factor ε_{t+1}. By formula (15), the correlation factor between $\ln m_{t+1}$ and r_{t+1} is equal to $sign(-\lambda)$ and

hence it is equal to one for all negative values of λ. Since this parameter does not belong to directly observable (measurable) variables, it will be called a latent parameter of the model under study which characterizes the investor attitude (or, as is usually said, the bond market) to the risk of variation in the short-term rate in the future period. The meaning of the latent parameter manifests itself most clearly in the formula of the forward short-term rate $f_t^1(n, n+1)$ connecting the prices of n- and $n+1$ - period zero-coupon bonds at time t

$$f_t^n(n, n+1) = \ln b_t^n - \ln b_t^{n+1}. \tag{18}$$

To form a notion such as the forward short-term (one-period) rate $f_t^1(n, n+1)$ in terms of n periods, consider the hypothetic buying and selling of n- and $(n+1)$-period zero-coupon bonds at time t. We divide the entire procedure into three steps.

Step 1. We sell an n-period bond at time t at the price b_t^n and buy several $(n+1)$-period bonds at the price b_t^{n+1} in the amount of $b_t^n \cdot \left(b_t^{n+1}\right)^{-1}$.

Step 2. We pay a money unit at time $t+n$ for an n-period bond.

Step 3. At time $t+n+1$, we obtain an income in the amount of $b_t^n \cdot \left(b_t^{n+1}\right)^{-1}$ money units for the bought ($n+1$)-period bonds. Thus, we have spent one money unit at time $t+n$ and obtained $b_t^n \cdot \left(b_t^{n+1}\right)^{-1}$ money units at $t+n+1$ time. This operation can be associated with the one-period yield $f_t^1(n, n+1)$ starting from the relation $1 \cdot \exp[f_t^1(n, n+1)] = b_t^n \cdot \left(b_t^{n+1}\right)^{-1}$ which implies formula (18).

According to (2)-(5), in the considered model of arbitrage-free pricing, the value of short-term forward rate can be represented as

$$f_t^1(n, n+1) = r_t + B_n \cdot (\theta - r_t) \cdot (1-\varphi) - w_0 \cdot B_n \cdot [\lambda + 0,5 \cdot B_n]. \tag{19}$$

The role of the parameter λ in the initial model is completely clarified precisely if we write down (19) in the form

$$\begin{aligned} &f_t^1(n, n+1) = E_t\{r_{t+n}\} - \\ &-w_0 \cdot (1-\varphi^n) \cdot (1-\varphi)^{-1} \cdot [\lambda + 2^{-1}(1-\varphi^n) \cdot (1-\varphi)^{-1}] \end{aligned} \tag{20}$$

The first term in (20) has the form of conditional mathematical expectation for the short-term rate over n periods and the second term can naturally be interpreted as the risk premium for the one-period rate over n periods. An analysis of formula (20) shows that the risk premium is positive for $\lambda < [-2 \cdot (1-\varphi)]^{-1}$ and the asymptotics of the short-term forward rate over a large number of n periods is greater than the average current short-term rate θ.

In this case, if $\lambda < -(1-\varphi)^{-1}$, then the difference $f_t^t(n, n+1) - E_t\{r_{t+n}\}$ is proportional to $w_0 = D_t\{r_{t+1}\}$ and monotonically increases with n.

The current maturing yields of the n-period bond (17) in the model under study is given by the formula:

$$\begin{aligned} Y_t^n = &(\theta - w_0 \cdot (1-\varphi)^{-1}(\lambda + 2^{-1} \cdot (1-\varphi)^{-1})) - w_0 \cdot (1-\varphi)^{-2} \cdot (2n)^{-1} \cdot BB_n + \\ &+ (r_t - \theta + w_0 \cdot (1-\varphi)^{-1} \cdot (\lambda + (1-\varphi)^{-1})) \cdot n^{-1} B_n \end{aligned} \tag{21}$$

The quantity Y_t^n monotonically increases with n in the range of the parameters

$$\theta - r_t > w_0 \cdot (1-\varphi)^{-1} \cdot (\lambda + (1-\varphi)^{-1}) \tag{22}$$

In this inequality, all parameters of the model except r_t are constants, but r_t is a time dependent variable. If the investors determine the risk parameter by the inequality

$$\lambda < -(1-\varphi)^{-1} - w_0^{-1} \cdot (1-\varphi) \cdot (\max_t r_t - \theta),$$

then this results in the monotone increase in the maturing yield of zero-coupon bonds in time not only for the mean yields $E\{Y_t^n\}$ but also for the current yields Y_t^n.

The preceding analysis of the influence of the latent risk parameter λ only allowed one to obtain several estimates from above in the situations that are often met on the market. Since in the model under study, the parameter λ is determined by investors, it can be interpreted as an adjustable parameter for matching the calculated arbitrage-free and actually observable yields of zero-coupon bonds for all maturity dates.

This program can easily be realized in our model. Consider the goal function $\Phi(\lambda)$ for adjusting the latent variable λ to the observed yield data y_t^n:

$$\Phi(\lambda) = \sum_{n=1}^{N} \sum_{t=1}^{T} (Y_t^n - y_t^n)^2 = \sum_{n=1}^{N} \sum_{t=1}^{T} \left(\frac{A_n + B_n \cdot r_t}{n} - y_t^n\right)^2 \quad . \tag{23}$$

Since Y_t^n is a linear function of the latent parameter λ, the function $\Phi(\lambda)$ is a positive definite quadratic function of λ whose minimal value is attained at $\lambda = \lambda^*$:

$$\lambda^* = w_0^{-1} \cdot (1-\varphi) \cdot (\mu_1 - \mu_2) - [2 \cdot (1-\varphi)]^{-1} \cdot (1-\mu_3), \tag{24}$$

where

$$\mu_1 = \theta - \frac{1}{T} \sum_{t=1}^{T} y_t^1, \quad \mu_2 = \sum_{n=2}^{N} h_n \cdot \frac{1}{T} \sum_{t=1}^{T} (y_t^n - y_t^1), \quad \mu_3 = \sum_{n=2}^{N} h_n \cdot n^{-1} \cdot (B_n - BB_n),$$

$$h_n = (1 - n^{-1} \cdot B_n) \cdot \left[\sum_{k=2}^{N} (1 - k^{-1} \cdot B_k) \right]^{-1}$$

Analysis of «MosPrime» Credit Rates

As a data example, it was considered the credit rates of the Moscow Market of Interbank Credits, MosPrime. The MosPrime Rate, i.e., the Moscow Prime Offered Rate, is the indicative rate of rouble credits (deposits) on the Moscow money market.

This indicator is formed by the National Currency Association (NCA) on the basis of the deposit rates of «overnight» terms of 1 week, 2 weeks, and 1, 2, 3, 6 months announced by 8 banks which are the leading operators on the Interbank Credit Market. In our notation, the symbols w1, w2, m1, m2, m3, and m6 denote the deposit rates in percents per annum for 1 week, 2 weeks, 1 month, 2 months, 3 months, and 6 months, respectively. All data are taken from the site http://www.nva.ru and are presented in the Appendix. These data refer to the period January-August 2013.

A credit contract on the interbank market can be considered as a zero-coupon bond, because it is standardized with respect to its volume and terms. These contracts are quoted according to their credit rates, but these quotations can easily be converted to the bond prices by using the price-rate dependence.

The weekly rate as a short-term rate was chosen. When choosing a week as the shortest duration, it is necessary that all of other durations were divisible by this shortest duration. For uniform harmonization of such requirements, the year was divided into 48 weeks, a month, into 4 weeks. The following initial data were used to construct the observable yields y_t^1:

$$y_t^1 = \ln(1+\frac{w1_t}{a}),\ y_t^2 = \frac{1}{2}\ln(1+\frac{2\cdot w2_t}{a}),\ y_t^4 = \frac{1}{4}\ln(1+\frac{4\cdot m1_t}{a}),\ a = 4800$$

$$y_t^8 = \frac{1}{8}\ln(1+\frac{8\cdot m2_t}{a}),\ y_t^{12} = \frac{1}{12}\ln(1+\frac{12\cdot m3_t}{a}),\ y_t^{24} = \frac{1}{24}\ln(1+\frac{24\cdot m6_t}{a}).$$

Naturally the value y_t^1 was taken as a short-term rate r_t used to construct regression (2). Table 1 presents the results of the regression construction.

Table 1. Results of the regression

Notation	**Estimate**	**T statistics**	**p-level**
φ	0,531604	4,54	0
$\theta\cdot(1-\varphi)$	0,000622	4,03	0
θ	0,001327		
w_0	1,10661E-09		

For 32 observations $R^2 = 0,41$, the White test does not disprove the variance homogeneity hypothesis and the t-statistic of the unit root for the equality $\varphi = 1$ is equal to minus 4,00, while the critical Dickey-Fuller statistic at the 5% significance level is equal to minus 2,93 for this model. Since the Durbin – Watson statistic value DW=1,98, which is close to 2, is not a reliable value in the autoregression models, it is necessary to present the data of the sample autocorrelation remainder function. The error autocorrelation tests (Ljung&Box statistics) show that the hypothesis H_0 of the error autocorrelations equal to zero cannot be rejected at the 5% level of significance.

The calculated optimal value for latent variable is $\lambda^* = -62295,6$; the value of the goal function is $\Phi(\lambda^*) = 2,07\cdot10^{-7}$. For comparison, the value of the corresponding «discrepancy» obtained in the construction of regression (2) is equal to $3,32\cdot10^{-8}$, and calculated for five yields, the value of the goal function is only 1, 24 times greater than the «discrepancy» for one short-term rate, which is a rather good result. The optimal value λ^* for the data under consideration corresponds to the following expression for the stochastic discount factor (14):

$$-\ln m_{t+1} = 2,147 + r_t - 2,072\cdot\varepsilon_{t+1}.$$

The general comparison picture of the calculated and observed yields for different durations of the loan represented in Fig. 1, where the horizontal axis represents the standard deviation and the vertical axis - the time average of the yields.

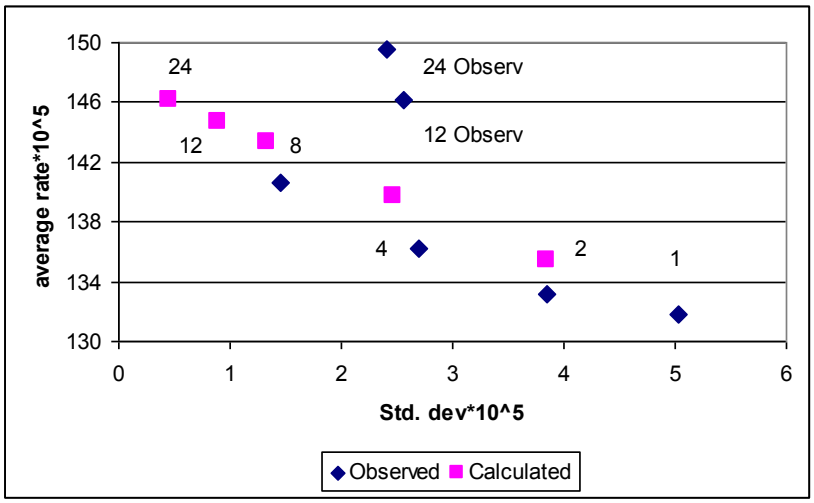

Fig. 1. The time averages and the standard deviations of the observed and calculated rates of return for different durations of the loan. Separate figures indicate the duration of the loan in weeks.

An analysis of the results showed the following. A steady increase in the rates of return with increasing duration of credit is observed in both the observed and calculated values. For a period of two, four and eight weeks, the calculated values exceed the average observed values (the relative excess is about 2%). On the contrary, for credit rate of return for periods of twelve and twenty four weeks, the observed values are higher than the calculated values with a relative error of 2%.

Conclusion

It was considered the term structure of credit rates on the Moscow Market of Interbank Credits in the period of nine months in 2013. It was shown that the application of the proposed model of arbitrage-free pricing of zero-coupon bonds to the MosPrime data of the credit market exhibits a good consistency with observations. A similar result was obtained with the data for 2010 (Ajevsky, V.V. & Chetverikov, V.M. (2012)). All results refer to short interbank loans up to six months.

In the proposed model, there is a latent free parameter characterizing the bonus for the risk per unit of volatility. This parameter is not directly measurable, and characterizes the attitude to risk all investors in general. The value of this parameter largely determines the temporal structure of interest rates. It can be interpreted as a *single* adjustable parameter for matching the calculated and actually observable yields *for all credit durations*. Additional studies show that the direct regression of n - week's rate on a one-week rate of return gives an unsatisfactory result.

Appendix

Table 2. Data of «MosPrime» credit rates

date	w1	w2	m1	m2	m3	m6
15.01.2013	5,58	5,96	6,48	6,98	7,38	7,63
22.01.2013	6,03	6,16	6,49	6,89	7,27	7,5
29.01.2013	6,25	6,28	6,45	6,81	7,1	7,34
05.02.2013	5,83	6	6,37	6,78	7,12	7,33
12.02.2013	5,76	5,97	6,28	6,75	7,13	7,32
19.02.2013	6,21	6,23	6,4	6,74	7,06	7,33
26.02.2013	6,4	6,35	6,4	6,71	7,04	7,31
05.03.2013	6,14	6,21	6,32	6,64	6,97	7,22
12.03.2013	6,36	6,41	6,5	6,72	7,15	7,31
19.03.2013	6,25	6,36	6,45	6,7	7,05	7,25
26.03.2013	6,34	6,37	6,5	6,72	7,09	7,32
02.04.2013	6,31	6,38	6,59	6,88	7,26	7,49
09.04.2013	6,3	6,37	6,53	6,82	7,24	7,49
16.04.2013	6,31	6,35	6,5	6,8	7,13	7,38
23.04.2013	6,45	6,46	6,51	6,8	7,16	7,4
30.04.2013	6,56	6,59	6,64	6,81	7,18	7,4
07.05.2013	6,54	6,61	6,65	6,8	7,18	7,4
14.05.2013	6,55	6,56	6,65	6,81	7,16	7,4
21.05.2013	6,56	6,6	6,72	6,84	7,19	7,34
28.05.2013	6,59	6,6	6,74	6,84	7,12	7,36
04.06.2013	6,58	6,6	6,74	6,83	7,09	7,33
11.06.2013	6,38	6,49	6,61	6,76	7,04	7,28
18.06.2013	6,46	6,52	6,7	6,83	6,99	7,23
25.06.2013	6,54	6,63	6,78	6,88	7,03	7,28
02.07.2013	6,59	6,64	6,76	6,88	7,03	7,25
09.07.2013	6,18	6,29	6,59	6,78	7	7,22
16.07.2013	6,44	6,5	6,7	6,82	7	7,2
23.07.2013	6,45	6,5	6,64	6,79	7	7,19
30.07.2013	6,5	6,5	6,64	6,77	7	7,2
06.08.2013	6,37	6,42	6,53	6,73	6,9	7,17
13.08.2013	6,23	6,33	6,49	6,69	6,88	7,14
20.08.2013	6,39	6,43	6,5	6,72	6,85	7,14
27.08.2013	6,41	6,46	6,49	6,7	6,8	7,05

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