Deindustrialization: "Vanishing Cities" Revisited*

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Abstract

Anas's (2004) impossibility theorem ("vanishing cities") states that monopolistic competition, economies of scale alone—are insufficient to explain growth of cities in response to growing population or decreasing trade costs; cities decline. Is this conclusion an artefact of unrealistic assumptions? Instead of Anas's normative approach with social planner, we consider positive approach: migration or developers' equilibria. Still, "vanishing" remains robust to more realistic modifications! Finally, we interpret "vanishing" as a realistic outcome: *industries*, *free of externalities*, *should locate in countryside*. Generalizing our comparative statics, we conclude that simultaneous growth of number and size of cities is implausible in such industries.

Key words: city sizes, growth, agglomeration, trade

JEL codes: D62, F12, R12

1 Introduction

Contemporary urban development shows rather typical tendency of many manufacturing facilities to locate in small cities or even in a countryside. The examples include not only traditional milling, beer brewing and other food industries but even car assembling. More generally, nowadays manufacturing is one of the least urbanized activities (see, for instance, Holmes and Stevens (2004) on U.S. and Canada or Kolko (2010) on comparison between manufacturing and services). As a result, big cities are more and more "deindustrialized", specialized on exporting services. The services are governance of territories by governments, governance of multi-plant firms by headquarters, education, banking, etc. Small cities, instead of exporting such services, produce manufacture and tend to decrease in size. Should we say that decreasing small cities can be a *natural* outcome of market evolution, driven by noticeably reduced trade costs?

Urban theory and economic geography (see Fujita and Thisse, 2002) has a lot to say about "agglomeration forces" driving firms and workers together into cities, and about countervailing "dispersion forces." Among the latter, commuting costs, land rent and other diseconomies of size understandably restrict the city size. Alternatively, well-known Krugman's Core-Periphery model uses agricultural population as a dispersion force instead of commuting costs. It predicts that large

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trade costs can support many small cities, whereas *decreasing trade costs force agglomeration into few large cities*, and this view has become popular.

The opposite tendency — evolution towards smaller and smaller cities — is also predicted by market theory in several settings. In particular, Starrett's (1978) impossibility theorem states that without economies of scale or externalities, explanation of trading cities is impossible: economy would generate "backyard capitalism. However, even accounting for externalities, Henderson (1974) has achieved somewhat similar prediction. He assumes intra-industry externalities, but inter-industry ones are absent, commuting costs work as dispersion force. Then, such world generates mono-industry cities. Describing competition between two cities or regions, Helpman (1998) has explored the tension between the agglomeration force stemming from preference for variety and a dispersion force stemming from limited housing supply. Somewhat similarly treating two cities, Tabuchi (1998) introduces competition for land as a dispersion force. Both models result in similar result on *arising dispersion*: equal distribution of population between two regions when transportation costs become low enough.

Extending this line of reasoning to emerging system of cities instead of given two cities, Anas (2004) studies a normative setting. World (or country) population given, a social planner maximizes the per-consumer welfare by choosing the number of cities, under monopolistic competition à la Dixit-Stiglitz, with iceberg trade costs. The agglomeration force amounts to economies of scale, whereas the dispersion force stems from commuting costs (a square root of the city size). The main theorem describes optimal cities under growing world population as follows: their number increases but the size of each *decreases*, and eventually *drops down* to technologically admissible minimum. The explanation is that the benefit of living in a big city (close to many producers) decreases when more and more varieties are imported from a growing world, whereas the commuting costs remain the same. Anas interprets his surprising result as a theoretical counter-example: without externalities simple monopolistic competition mechanism is *insufficient to produce growth of cities* in response to growing population, or/and decreasing trade costs, to understand the evolution of cities. Our goal is to check robustness of this idea and to expand it.

Our setting uses Anas's (2004) model as a baseline. We have suspected his normative setting and restrictive assumptions to drive his unexpected result. Does it remain valid in more realistic settings? Instead of central planner, we explore two positive alternatives: (1) migration equilibrium, where each citizen voluntarily can choose a city to live or can settle a new city (understanding how production and trade will respond to her choice); (2) developers' equilibrium, where each city decides to invite or not additional citizens (also understanding the production/trade consequences). These two versions remind two cases in theory of clubs: the migration setting describe "open clubs" that everybody can joint irrespective of the will of its residents; another one relate to "closed clubs."[1]

As in Anas (2004), we use familiar Dixit-Stiglitz preferences and iceberg trade costs, assume some technological minimum of city size called "village." Commuting costs also keep the same form and we similarly stick to symmetry, ignore the discrete nature of cities, firms and citizens, continuous model being more tractable.

Among results, we start with *Migration equilibria.* These equilibria are multiple, not uniquely determined by preferences and costs. One can study only the "zone of equilibria." Formally, under given world population L and trade freeness ϕ, every symmetric composition of n cities (of equal sizes) is a migration equilibrium, in the sense that consumer's utility is the same everywhere.

[1] We cannot directly rely on club theory, because our clubs-cities are *interacting*: they influence each other through trade.

However, we consider only *stable* equilibria. Stability means that small perturbation should be reversed by migration forces. It includes two conditions: (i) no dispersion: nobody wants to settle a new city; (ii) no agglomeration: nobody wants to migrate to another similar city (which thereby could become slightly bigger and less attractive).

Propositions 1 and 2 describe "vanishing" cities. The zone of (L, ϕ, n) admissible for stable cities turns out bounded, cities must disappear in 3 cases: (i) when the current number n of cities becomes bigger than certain uniform bound n^*, or (ii) when world population L becomes bigger than certain uniform bound L^{d*}; or (iii) when trade freeness ϕ becomes bigger than certain uniform bound ϕ^{d*}.

It means that, other parameters given, whatever the historical city system is, growth of L, ϕ or n makes our city system abruptly switching to complete dispersion, because of individual migration. This conclusion on migration looks more interesting for us than Anas's global optimality of cities. It surprisingly contrasts Krugman's agglomeration outcome. Why? Because Krugman's agglomeration force and dispersion force decrease together with trade freeness, and the second force decreases faster, which seems a doubtful assumption for modern cities. In our case, as in Krugman, *the agglomeration force is the benefits of combining firms and consumers, it decreases in trade freeness. However, our dispersion force is the commuting cost and it does not change with trade freeness, this explains the outcome.* Similarly, under constant city sizes, the dispersion force does not change with world population but the agglomeration force weakens, because of increasing share of imported varieties in consumption, that makes domestic production less important.

Of course, both Anas's and our analysis of this general tendency are too stylized, in comparison to richer urban models that describe a heterogenous city system, for instance, Tabuchi and Thisse (2006), Behrens et al. (2010). We rather provide a link between these developed models and two-regional models that economic geography has relied on. Our goal here is more modest; we just make a step in analysis of endogenous city systems and show that some industry may become dispersed, due to the tendency revealed by Anas. We also interpret this as a realistic outcome for some kind of cities, in contrast to his negative interpretation.

However, instead of limiting cases, detailed comparative statics can be more interesting: What happens to the city system before its abrupt disintegration into villages? How economic parameters matter for *gradual* changes in cities? Proposition 3 states that growing population the migration stability can make city size either gradually shrink or collapse but not increase.

To get additional predictions, one should cope with equilibria multiplicity and define a reasonable *selection* among equilibria. Our "developers' equilibrium" imposes new restrictions on cities, in addition to migration stability. We assume that citizens are able to restrict the entry to their city, or attract new people by small privileges, and thereby increase average welfare. Such collectively rational behavior is represented by benevolent city government called "local developer or city major" (unlike Anas's global planner but alike those developers who maximize the price of land by trying to please the citizens). Such developers may launch a new city. When attracting a citizen, the developer is myopic enough to ignore the consequences of this development to the whole world: depopulating other cities and changing their economic variables. Caring only about changes in her city, correctly anticipating production and trade, she optimizes average welfare with respect to her city size N. We explore two versions of such equilibria: "myopic" or "wise", both displaying similar features.

It turns out that the zone of (symmetric) stable developers' equilibria is a curve $N(L)$ within migration equilibria, it is bounded near the origin, for higher L it disappears and the result is cities' disintegration (Proposition 4). Additionally (whenever cities are multiple), the equilibrium city size *gradually decreases* with world population or trade freeness, before dropping down to its technological minimum. This version of the model differs from Anas's global optimization—in respect of ignoring the interests of other cities by local government and her myopic views on the rest of the world. However, the main conclusion remain: the tendency of (manufacturing, industry-specific) towns to decrease and eventually collapse down to minimal technological size.

Summarizing, we generally support this prediction and conclude that *cities comprised of industries lacking intra- or inter-industries externalities—should have a tendency to decrease and eventually reach their minimal technological size.* The rest of the paper is organized as follows: Section 2 presents our baseline model, Section 3 studies migration equilibria. Section 4 deals with developers' equilibria, and the final section concludes. The Appendix is devoted to proofs and possible extension of the model.

2 Model: system of cities with migration

Now we introduce the model of cities' system very close to Anas's one, except for: (1) possibly asymmetric cities and (2) migration process instead of social planner. We start with the description of internal city structure and then embed it into a system of cities. We construct the general equilibrium one sector model and relegate to the Appendix B the discussion of the possible extension of the model to two sectors.

City. Traditionally, we consider monocentric and circular cities endowed with Central Business District (CBD) where the production and trade take place. The only production input is labor supplied by consumers – citizens. Each consumer needs one unit of land and possesses a unit of time which she spends on commuting to the workplace (CBD) and labor. The cost of commuting is s units of time per a unit of distance, therefore, consumer living at distance x from CBD spends sx units of time for commuting and supplies $h(x) = 1 - sx$ units of labor for production. Suppose there is N residents in a given city. Then, the radius of such a city becomes $r = \sqrt{N/\pi}$. Given individual labor supply and uniform distribution of citizens within a city, overall labor supply for production H is given by

$$H(N) = \int_0^r 2\pi x h(x) dx = \pi r^2 - 2\pi r^3/3 = N - kN^{3/2}, \qquad (1)$$

where $k \equiv 2s/3\sqrt{\pi}$ is a constant, summarizing commuting cost. We denote the average (per-citizen) labor supply in a city as

$$\theta(N) \equiv H(N)/N.$$

In addition, we assume zero opportunity value of land and free reallocation within a city.

Now, we explain how redistribution of rent makes income proportional to the labor supply. Suppose that the city size is r. We have normalized the land rent on the edge of the city to zero. Since consumers face the same price vector, free reallocation of consumers should lead to disposable income equalization among them. In addition we assume that the local government collects the land rent and distributes it equally among citizens in a form of lump-sum transfer. Then disposable income of every citizen in a city of size N is

$$I(N) = \theta(N)w \equiv (1 - k\sqrt{N})w,$$

which is decreasing in city size.[2]

[2] To see details of rent redistribution, see that at any city point the sum of rent cost and commuting cost must be

System of cities and goods. Suppose there are n cities with population masses $(N_1, N_2, ..., N_n)$. There is only one differentiated good in the economy. Each variety is produced by only one firm residing in some city. All cities trade to each other through some common "hub", which can be just sea, i.e., transport cost for each pair of cities is the same. The market for varieties is monopolistically competitive. So, endogenous is the mass m_i of varieties (and hence, firms) produced in city i.

Consumers have identical Constant Elasticity of Substitution (CES) preferences over the set of varieties:

$$U = \left[\sum_{i=1}^{n} \int_0^{m_i} x_{kji}^{(\sigma-1)/\sigma} dj \right]^{\sigma/(\sigma-1)}, \tag{2}$$

where x_{kji} is a consumption of variety j produced in the city i by a consumer in the city k and $\sigma > 1$ is the elasticity of substitution. Consumer k maximizes in x_k her utility (2) subject to the budget constraint

$$\sum_{i=1}^{n} \int_0^{m_i} p_{kji} x_{kji} dj \le I(N_k), \tag{3}$$

where p_{kji} is the price of variety j produced in city i and consumed in city k. Taking the first-order conditions and expressing the Lagrange multiplier from the budget, we obtain the consumer demand function $\mathbf{x}(\cdot)$ in the form:

$$\mathbf{x}_{kji}(p_{kji}, I_k, P_k) = p_{kji}^{-\sigma} I(N_k)/P_k^{1-\sigma} \qquad P_k = \left[\sum_{i=1}^{n} \int_0^{m_i} p_{kji}^{1-\sigma} dj \right]^{1/(1-\sigma)}, \tag{4}$$

with P_k being a price index. It is "perfect", as a price of one unit of utility, in the sense that indirect utility of a consumer in the city k is $V_k = I(N_k)/P_k$.

In Appendix F we also explain the two-sector version of the model, where preferences over the two goods have standard Cobb-Douglas form:

$$\check{U} = \left(\left[\sum_{i=1}^{n} \int_0^{m_i} x_{kji}^{(\sigma-1)/\sigma} dj \right]^{\sigma/(\sigma-1)} \right)^{\mu} M^{1-\mu}.$$

As soon as the budget shares remain constant, the qualitative features for this extension remains the same as the below analysis of one-sector model. The extension explains how manufacturing cities can coexist with services-cities, being important for interpretation.

Producer. Each producer is a price-maker for her variety. Standardly for monopolistic competition literature, we assume that producer in city i has a cost function $C(y) = (cy + F)w_i$, i.e. producer has fixed labor cost F to set up a plant and marginal labor requirement c of production. Trade within a city is costless, whereas trade with other cities requires iceberg transportation cost $\tau > 1$. It means that supplying one unit of good in city k requires τ units of good shipped from city i. Under these assumptions profit function of each producer j in city i is

$$\pi_{ji} = \sum_{k=1}^{n} (p_{kji} - \tau_{ki} c w_i) \mathbf{x}_{kji}(p_{kji}, I_k, P_k) N_k - F w_i \tag{5}$$

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the same. Hence, the rent at any point x must be $R(x) = s(r - x)w$, where w is wage per unit of time, and total rent in the city is $TR = \int_0^r 2\pi x s(r - x) w dx = \pi s w r^3/3 = kwN^{3/2}/2$. Since local government distributes the rent equally, we can now calculate disposable income of every citizen in a city of size N as $I(N) = w(1 - 3k\sqrt{N}/2) + kwN^{3/2}/2N = (1 - k\sqrt{N})w$.

which she maximizes in prices subject to demand functions (4) taking the price indexes as given. Here, $\tau_{ki}=1$ if $k=i$ and $\tau_{ki}=\tau$ otherwise, reflecting the fact that consumption in a city different from our city requires larger production. Standardly, under CES preferences, such profit function is concave and has a unique maximum. Then, symmetry of producers leads to *symmetric* pricing by all firms from a given city. This allows us to drop index j in further discussion. Standard first order condition for producer's problem delivers

$$p_{ki}=\frac{\sigma\tau_{ki}cw_i}{\sigma-1}\qquad \pi_i=\left(\sum_{k=1}^{n}\frac{\tau_{ki}cx_{ki}N_k}{\sigma-1}-F\right)w_i. \tag{6}$$

Free entry into the market drives firms' profit in every city to zero. Combining zero-profit condition with labor market clearing yields equilibrium mass of varieties in every city:

$$m_i=\frac{N_i\theta(N_i)}{F\sigma}\quad \forall i. \tag{7}$$

Finally, equilibrium wages and their consequences $p_{ki}, I(N_k)$ can be obtained from market clearing for a representative variety produced in each city:

$$\sum_{k=1}^{n}\frac{\tau_{ki}p_{ki}^{-\sigma}I(N_k)N_k}{P_k^{1-\sigma}}=(\sigma-1)F/c\quad \forall i \tag{8}$$

Trade equilibrium associated with a system of n cities of given sizes $(N_1,N_2,...,N_n)$ is defined as a bundle $\{x_{ki}\}_{i=\overline{1,n}}^{k=\overline{1,n}}$ of consumption values, a bundle of prices $\{p_{ki}\}_{i=\overline{1,n}}^{k=\overline{1,n}}$, vector of varieties masses $\{m_i\}$ and vector of wages $\{w_i\}$ such that: (1) consumption values solve consumers problems (2) subject to budget constraint (3) given prices, wages and set of available varieties; (2) prices solve producers problems (5) given demand function (4), price indexes and wages; (3) firms earn zero profit (free entry); (4) labor market and market for every variety clear.

We do not discuss existence of such general equilibria, pointing out later on existence of symmetric ones (that we study and disturb). Further, every trade equilibrium delivers indirect utility $V_k=\theta(N_k)w_k/P_k$ to any consumer in city k. Suppose, the world population amounts to L consumers.

Standardly, we call n cities of sizes $(N_1,N_2,...,N_n)$ a **migration equilibrium** if (1) $\sum_{i=1}^{n}N_i=L$ and (2) related trade equilibrium yields the same level of indirect utility across cities: $V_k=V_i\quad\forall k,i$.

Naturally, the symmetric distribution of population across arbitrary number of cities is a migration equilibrium. Indeed, symmetric cities imply symmetric trade equilibrium, then cities are interchangeable. However, our goal is to understand, when such symmetric migration equilibrium is *stable*, in the sense that small perturbation is not amplified. Therefore, we shall consider mainly symmetric (or close to symmetric) population distributions across cities. Let us reserve notation (n,N) for the symmetric equilibrium with n cities of size N, so that $L=nN$.

3 Migration stability

In this section we discuss the stability of any symmetric equilibrium (n,N) against small perturbation in population distribution. First, we define two kinds of stability: migration to villages and migration to other cities.

1. Consider a system of slightly asymmetric cities. Starting from n cities of size N, suppose that new $(n+1)$-st city of size ε is created, one of old cities taking size $\tilde{N} = N - \varepsilon$. If in new trade equilibrium indirect utilities V_i evaluated at point $\varepsilon \approx 0$ satisfy $V_{n+1}(\varepsilon) < V_n(\tilde{N})$, we say that this migration equilibrium (n, N) is (strictly) **stable against dispersion**, otherwise it is not.

2. Consider a system: 1-st city of size $N_1 = N + \varepsilon$, 2-nd city of size $N_2 = N - \varepsilon$ and $n-2$ cities of size N. If in related trade equilibrium incremental utility $\frac{dV_1(N+\varepsilon)}{d\varepsilon}$ evaluated at the point $\varepsilon \approx 0$ is negative, we say that migration equilibrium (n, N) is (strictly) **stable against agglomeration**, otherwise it is not.

When a symmetric migration equilibrium (n, N) satisfy both stability conditions, we call it a **migration stable equilibrium**.

The first definition means that small shift of population from a city into previously unpopulated area doesn't create incentives for catastrophic movement to this newly created town. The second notion of stability means that small movement of population from one city to another does not make this city more attractive, it makes sense when $n \geq 2$.

Now we formulate the conditions when a symmetric equilibrium is stable in both senses.

Lemma. (Stability conditions) *(1) Symmetric migration equilibrium* (n, N) *is stable against dispersion if and only if*

$$1 - k\sqrt{\frac{L}{n}} > \left[\frac{1 + (n-1)\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right]^{-\frac{1}{\sigma} - \frac{1}{\sigma-1}}. \tag{9}$$

(2) Symmetric migration equilibrium (n, N) *is stable against agglomeration if and only if*

$$\frac{2\sigma - 1}{(\sigma - 1)\left(\sigma - 1 + \sigma\frac{1+(n-1)\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\right)} < \frac{k\sqrt{L}}{2\sqrt{n} - 3k\sqrt{L}}. \tag{10}$$

Proof. See Appendix.

□

The region of stable combinations (L, ϕ, n) described in Lemma is displayed on Fig. 1 for specific values $\sigma = 11$, $k = 0.005$ (simulated). *All* combinations inside this shaded area generate stable equilibria, because our Lemma gives necessary and sufficient conditions.

In Fig. 1 we see that the zone of stable equilibria is *bounded* in all three dimensions L, n, ϕ. This zone has a complex shape: there is a grotto under the hill (see our subsequent figures for details). Our further plan is to prove such boundedness for *any* parameter values k, σ.

In some sense, it means studying the comparative statics of such shaded area. Specifically, to correctly resolve the Anas's question about "vanishing cities", we describe now, how the *region of stable migration equilibria* changes with the population of the whole system or/and with trade frictions.

First of all, we simplify notation by (standardly) introducing trade freeness $\phi \equiv \tau^{1-\sigma} \in [0, 1]$. It is decreasing in the elasticity of substitution σ and higher ϕ imply freer trade. For any number n of cities, the condition of stability against dispersion can be reformulated in two alternative ways:

(1) given trade freeness ϕ, total population is bounded from above as

$$L(n) \leq L^d(n) = \frac{n}{k^2}\left[1 - \left(1 + \frac{1-\phi}{n\phi}\right)^{-\frac{1}{\sigma} - \frac{1}{\sigma-1}}\right]^2;$$

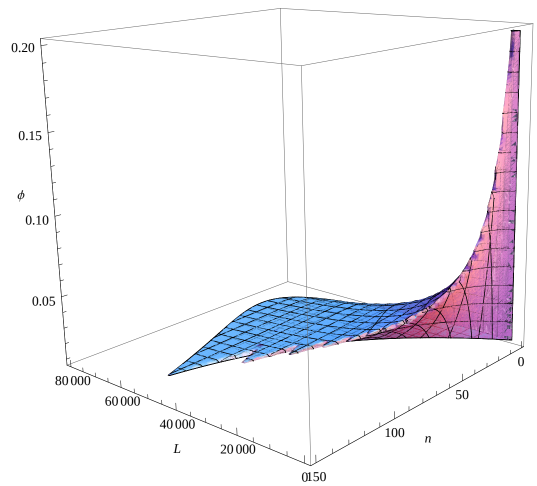

Figure 1: Region of stable equilibria.

(2) given total population L, freeness of trade is bounded from above as

$$\phi(n) \le \phi^d(n) = \frac{1}{1 + n\left[\left(1 - k\sqrt{L/n}\right)^{-\frac{2\sigma-1}{\sigma(\sigma-1)}} - 1\right]}.$$

Similarly, under any number of cities n, the condition of stability against agglomeration requires that:

(1) given freeness of trade ϕ, total population is bounded from below:

$$L(n) \ge L^a(n) = \frac{n}{k^2}\left[\frac{2}{\sigma + 2 + \frac{(\sigma-1)\sigma n\phi}{(2\sigma-1)(1-\phi)}}\right]^2;$$

(2) given total population L, freeness of trade is bounded from below:

$$\phi(n) \ge \phi^a(n) = \frac{1}{1 + n\frac{\sigma(\sigma-1)}{(2\sigma-1)\left(\frac{2\sqrt{n}-3k\sqrt{L}}{k\sqrt{L}} - \sigma + 1\right)}}.$$

In other words, two kinds of stability conditions provide upper and lower bound on such parameter values that any symmetric migration equilibrium can be stable. Using these bounds, the following proposition shows that population growth or trade liberalization (increasing freeness) must lead to non-existence of stable equilibria.

Proposition 1. (No stable cities in large/free world)

(1) Maximal stable population $L^d(n)$ is bounded from above; i.e., there exists such $L^{d*} < \infty$, that any equilibrium $(n,\ L/n)$ is unstable against dispersion whenever $L > L^{d*}$;

(2) Maximal stable freeness $\phi^d(n)$ is separated from one; i.e., there exists such $\phi^{d*} < 1$, that any equilibrium $(n,\ L/n)$ is unstable against dispersion whenever $\phi > \phi^{d*}$.

Proof. (1) We start with the behavior of $L^d(n)$ when n goes to infinity. Brief inspection reveals that this limit is of type $\infty \times 0$ indeterminacy. However, we can apply l'Hospital's rule to rearrangements:

$$\lim_{n\to\infty}\left[k\sqrt{L^d(n)} = \frac{1 - \left(1 + \frac{1-\phi}{n\phi}\right)^{-\frac{2\sigma-1}{(\sigma-1)\sigma}}}{1/\sqrt{n}} = \frac{\frac{2\sigma-1}{\sigma(\sigma-1)}\left(1 + \frac{1-\phi}{n\phi}\right)^{-\frac{2\sigma-1}{(\sigma-1)\sigma}-1}\left(-\frac{1-\phi}{n^2\phi}\right)}{-\frac{1}{2n^{3/2}}}\right],$$

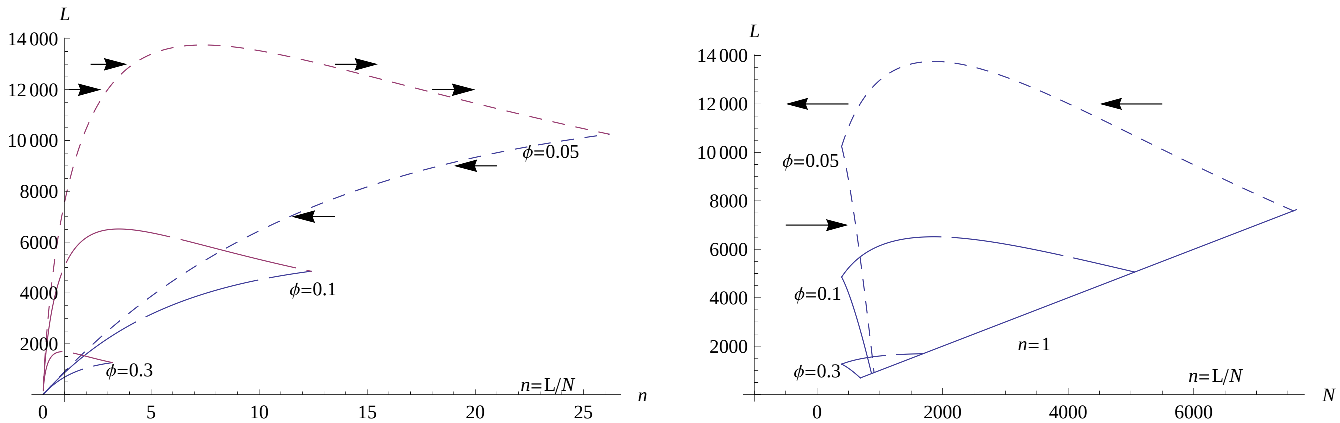

Figure 2: How region of migration stable equilibria shrinks w.r.t. trade freeness ϕ.

and get

$$\lim_{n\to\infty}\left[k\sqrt{L^d(n)}\right]=0.$$

Then, by continuity $\lim_{n\to\infty} L^d(n)=0$. This fact can be interpreted as existence of some $\bar{n}$ such that $\forall n\geq\bar{n}\quad L^d(n)\leq L^d(1)>0$. By Weierstrass theorem, $L^d(n)$ on interval $[1,\bar{n}]$ attains its maximum L^{d*} (which is finite) and, therefore, it is bounded by the value of this maximum on the whole interval $[1,+\infty)$. Then, for L bigger than the universal critical population L^{d*}, the equilibrium (n,N) is unstable for all n.

(2) The proof of the second part is similar. First, applying l'Hospital's rule to the expression for $\phi^d(n)$, it is possible to show that $\lim_{n\to\infty}\phi^d(n)=0$. Further, $\phi^d(n)$ is separated from one for any final n, and attains some maximum $\phi^{d*}<1$ at some finite n, like in previous argument. Thus, ϕ^{d*} is separated from 1, so that on the entire interval $[1,+\infty)\ni n$ any equilibrium $(n,\ L/n)$ is unstable against dispersion. □

In other words, *if world is large enough or trade is free enough, the only stable outcome is dispersion of population* to "villages".

The remaining question is boundedness of the region of stability (Fig.1) in dimension n. In other words, we ask whether there exist stable symmetric equilibria with large number of cities n. The next proposition precludes this possibility and, therefore, gives additional credibility for "vanishing cities".

Proposition 2. (No stable equilibria with many cities). *Under any admissible parameters (L,ϕ,k,σ), there exist some $\bar{n}$ such that any equilibrium with bigger number $n>\bar{n}$ of cities is unstable.*

Proof. See Appendix. □

Comparative statics. How the system of cities may *changes* when population grows or trade cost decreases? Do the cities in our model grow, gradually decrease or collapse? We first explain numerical simulations, interpret them, then develop them into a proposition.

Figure 2 plots the regions of migration stable equilibria in (n,L) coordinates for changing trade freeness: $\phi=0.05,\ 0.1,\ 0.3.$[3] In essence, the left panel of Fig.2 contains some projections of stable

[3]One can easily obtain that our stability conditions (9) and (10) can be rewritten as $L>\tilde{L^a}(N)=\frac{(2\sigma-1)(1-\phi)N}{\sigma(\sigma-1)\phi}\left[\frac{2}{k\sqrt{N}}-\sigma-2\right]$ and $L<\tilde{L^d}(N)=\frac{(1-\phi)N}{\phi}\frac{1}{(1-k\sqrt{N})^{\frac{\sigma(1-\sigma)}{2\sigma-1}}-1}$. It also follows from them that minimal stable city size does not depend on trade freeness.

zone from Fig.1 onto (n, L) plane. The right panel inverts this zone into (N, L) space, its straight line cuts away the cases with less than one city, alike vertical line with abscissa 1 in the left panel. Main observation is that *the larger is freeness—the smaller in some sense is the zone of stable equilibria*. Namely, the boundaries of the zone shrink towards the origin, making the stable area smaller.

First consider the direction of changes outside stability zone: Does the family of cities go towards stability or towards collapse? The arrows outside the zone show the migration tendency. In the right panel we observe that above the upper boundary the tendency works to *decreasing* size N of a city (which is shown in the left panel as increasing $n = L/N$). The arrow in the right side of the right panel says that when too-big unstable city looses its population, it can reach sooner or later the region of stability. Similar stable result is shown by the lower left arrow, which is *below the kink* breaking the left boundary. Instead, the upper-left arrow (*above the kink*) in this panel says, that under sufficiently high population too-small city further looses its population and *collapses* into a village. This (upper left) boundary of the stable zone is unstable.

Having this in mind, let us use this figure to express our intuitions about possible changes in the city system. To grasp possible impact of growing population L on cities under given ϕ, consider the following thought experiment. Assume, for instance, $\phi = 0.3$. Suppose we start with only one settlement ($n = 1$) and with population $L = 2$ which is small enough: Adam and Eve. What happens? This historical point of urbanization lies below the critical value $L^a(2)$, i.e., *below* the lower bound of related stability zone. So, it cannot happen that the couple lived apart, in different villages. Instead, they must agglomerate. Suppose now that population grows. Then the agglomeration tendency remains: all live in the same city. The picture tells that this single-city pattern of urbanization will persist until population exceeds approximately 600. Starting from this point further, our growing population can either remain in this city or try to settle a new one. Two-cities equilibrium becomes possible when population reaches approximately 1100. However, when population exceeds 1900, any n-city system looses stability, it abruptly collapses into villages consisting of 1 citizen each (minimal technological size).

Observe that different levels of trade cost make qualitative difference. Under $\phi = 0.1$ or $\phi = 0.05$, on the upper border of stability region there is a weak possibility that growing population may result in *gradually growing number of cities* instead of abrupt dispersion, at least on ascending wing of such region (and decreasing size of each city). Generally, under typical parameters, this fairy tale and related picture support the idea that *either gradual or abrupt decrease in city sizes* is possible in response to growing population or/and trade freeness. (Recall that we study only the cities based on increasing-returns reason for agglomeration, and leave aside other kinds of cities.)[4] We formulate now such tendency as a proposition.

Proposition 3. *Assume that during growth of population under fixed other parameters, the number of cities remain stable until the system reaches the border of stability zone, then this border governs the process. In this case, further evolution can display either gradually decreasing city size or abrupt collapse, but not an increase.*

Proof. In our stability condition (9), as well as in Fig.2, we see that it is the dispersion condition (not the agglomeration one) that limits the stable size of the world from above. Therefore, this

[4] On the other hand, when trade cost is large, after population hits level $L^d(1)$ there is a possibility of the creation of the another city and the existence of migration stable equilibria with more than one city and total population larger than $L^d(1)$. Nevertheless, straightforward inspection reveals that maximum stable city size $\frac{L^d(n)}{n}$ is decreasing in n, therefore, we conclude that it is implausible to observe increase in the representative city size with increase in the system population in the model economy.

condition rules the comparative statics. Its violation triggers the increase in the number of cities. From the formula we see that related city size $L^d(n)/n$ is a decreasing function. □

4 Developers' stability

Although notion of migration stability allows us to reduce the number of plausible equilibria and ensures our impossibility results, the remaining multiplicity of stable equilibria is somewhat disturbing. Therefore, in this section we develop another notion of stability as a selection criterion: stability against actions by a "developer" (local government) who aims to maximize the representative citizen's utility in her city and has some power to invite or push out citizens. This situation differs from simple migration by considering intra-city benefits from inviting a new citizen or forcing some citizen to go out. Previously we looked on this personal move from the migrant's point of view. Now we look from the side of the incumbents. Nevertheless, we still impose on the equilibrium the requirement of stability against dispersion (creation of a new city).

Wise developer. We assume in this paragraph that each developer completely predicts all changes in the trade equilibrium that will happen after inviting a new citizen from some other city.

(Symmetric) *wise developers' equilibrium* is a system of cities (n, N) such that it is a strict local Nash equilibrium among n developers choosing their city sizes. It means that there is an $\hat{\varepsilon} > 0$ such that all possible local ε-perturbations ($\varepsilon < \hat{\varepsilon}$) bring strictly negative benefit to a developer.

This notion does not touch the possibility of new cities and other asymmetric situations. By our assumption of wise predictions, the changes in the equilibrium trade and welfare coincide with predictions that we made when studying migration; the same equations remain valid. This allows us to show, that the new concept of equilibrium is a selection from previous concept. Namely, under any ϕ it is the lower border of related equilibria zone displayed in Fig. 2. Interestingly, a wise developer would chose a system with the largest number of cities and the smallest city size among migration stable equilibria.

Indeed, if the migrant goes out and thereby *decreases* welfare in the destination city, by symmetry, she *increases* welfare in the city of origin. Only when the derivative of welfare w.r.t. migration is zero, the situation can be a wise developers' equilibrium. Thus, we come to

Proposition 4. (Wise developers' equilibrium) *(i) A system of cities (n, N) is a wise developer's equilibrium only if it satisfies stability against agglomeration as equality:*

$$L = L^a(n) = \frac{n}{k^2}\left[\frac{2}{\sigma + 2 + \frac{(\sigma-1)\sigma n\phi}{(2\sigma-1)(1-\phi)}}\right]^2; \tag{11}$$

(ii) It belongs to migration-stable equilibria. Thereby, the developer's equilibrium remain possible within same three bounds: world population, trade freeness and number of cities—all must be small enough.

Proof. Established in text. □

Myopic developer. Although the introduced notion is perfectly rational, we find our requirements on local government's foresight too demanding. Indeed, the setup requires a developer to predict changes not only in his city, but in all cities throughout the country as well. To relax this requirement we introduce notion of myopic developer. More precisely, we assume that each developer is myopic (boundedly rational) in predicting outside trade consequences caused by excluding or inviting a citizen. It means that when maximizing welfare in her city, she expects no response from all relevant variables in other cities; population, price indexes and wages are taken as given. Suppose there are $n-1$ cities of size N, whereas #1 developer's city has size N_1 (to be optimized). Then, given symmetry in $n-1$ other cities, the (trade) equilibrium conditions for price index and wage in developer's city can be formulated as:

$$P_1^{1-\sigma} = \frac{N\theta(N)}{F\sigma}\left(\frac{\sigma\tau c}{\sigma-1}\right)^{1-\sigma}(n-1) + \frac{N_1\theta(N_1)}{F\sigma}\left(\frac{\sigma c w_1}{\sigma-1}\right)^{1-\sigma} \tag{12}$$

$$\frac{(n-1)\tau\theta(N)N[\sigma\tau c w_1/(\sigma-1)]^{-\sigma}}{P_i^{1-\sigma}} + \frac{\theta(N_1)N_1 w_1[\sigma c w_1/(\sigma-1)]^{-\sigma}}{P_1^{1-\sigma}} = (\sigma-1)F/c \tag{13}$$

This form of equilibrium conditions is standard. However, developer's optimization with respect to N_1 is different, since she takes N and P_i as given. Denote (perceived by developer) elasticities of price index and wage in city #1 with respect to local population N_1 as $\varepsilon_d^{P_1}$ and $\varepsilon_d^{w_1}$, respectively.

Definition. We call a symmetric equilibrium (n, N) *stable against myopic developer* if equal to zero is (perceived by developer) elasticity of indirect utility $\varepsilon_d^{V_1} \equiv \varepsilon^{\theta} + \varepsilon_d^{w_1} - \varepsilon_d^{P_1} = 0$ evaluated at $N_1 = N$, and the second derivative of the indirect utility w.r.t. N_1 is negative and the equilibrium stable against dispersion.

Proposition 5. (Myopic developers' equilibrium) *(i) A system of cities (n, N) is a myopic developer's equilibrium only if it satisfies the following condition:*

$$L = L^m(n) = \frac{n}{k^2}\left[\frac{2}{\sigma+2+\frac{(\sigma-1)\sigma n\phi}{(2\sigma-1)}}\right]^2; \tag{14}$$

(ii) It belongs to migration-stable equilibria. Thereby, the developer's equilibrium remain possible within same three bounds: world population, trade freeness and number of cities—all must be small enough.

Proof. See Appendix. □

The established condition for stability against myopic developer's action is similar to the condition of stability against agglomeration 10. It differs only by multiplier $(1-\phi)$ in the last term of the denominator. Thus, the costlier the trade is, the more developer stability behavior resembles that of stability against agglomeration (and of wise developer). The intuition is straightforward: cities affect each other through trade only. Therefore, the higher are trade cost, the less impact developer's city has on other cities, and the smaller is the developer's mistake in assumption of absent change in other cities. Moreover, the developer's myopia pushes the system towards fewer cities, i.e, larger size: $L^m(n) > L^a(n)$.

Now we present comparative statics analysis of the stability against developer's action graphically. Fig.3 is Fig.2 supplemented with the line of developer's stable equilibria $L^m(n)$ and its counterpart $\tilde{L^m}(N)$[5].

Observe that when the world population L grows, related point on the solid curve of developer's equilibria moves to the right in the left panel. Its counterpart shifts to the left in the right panel, which describes the same equilibrium in terms of the city size $N = L/n$. Such behavior means that *the number of cities increases in response to population growth*, whereas *the city size decreases.*

[5]Again one can easily derive that $\tilde{L^m}(N) = \frac{(2\sigma-1)N}{\sigma(\sigma-1)\phi}\left[\frac{2}{k\sqrt{N}} - \sigma - 2\right]$.

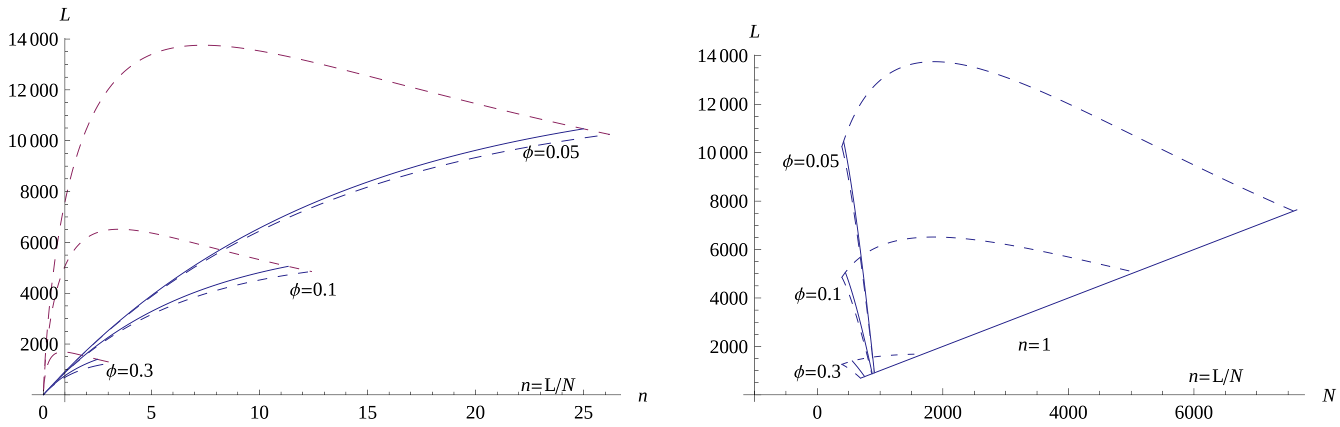

Figure 3: Developer stable configurations in coordinates (n, L) or (N, L) (under $k = 0.001$, $\sigma = 11$).

Similar conclusion follows for trade freeness, only the comparison goes not along each solid curve but comparing three curves. When freeness increases, the point of equilibrium (for any size of the world L) goes to the right in the left panel and to the left in the right panel. It again means that *the number of cities increases in response to decreasing trade costs*, whereas *the city size decreases*. Indeed, $L^m(n)/n$ is again decreasing function of ϕ meaning that new equilibrium must have lower city size and, hence, larger number of cities.

5 Conclusion

We have revisited Anas's theorem that claims monopolistic competition being insufficient to explain cities: in response to growing population or decreasing trade costs cities in his model decline and disappear. Questioning Anas's assumptions, instead of his normative approach, we consider migration or developers' equilibria. Still, *vanishing effect turns out robust* to these realistic modifications of the model, as well as to multiple sectors in the economy. It can be added that in another unpublished paper we have tried other forms of preferences and found that vanishing effect remains robust in this dimension also.

As a result, we came to interpretation of vanishing effect as a realistic outcome. In real life, many industries that *do not need externalities* from a city—really relocate to countryside or small towns. This model is one of possible explanations why it can be the case. Moreover, the two-sector model version suggests that industries with smaller shares in consumer spending—should be less urbanized; this hypothesis looks interesting and can be tested on data.

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A Proofs

Proof of Lemma. (1) Consider a city system $(n+1, N, N, ..., N-\varepsilon, \varepsilon)$ perturbed by a new town of size ε. Since first $n-1$ cities are symmetric we concentrate on trade equilibrium which is symmetric for those cities and take labor in the first city as numeraire. This implies wage $w_i = 1$ for all these $i = \overline{1, n-1}$. Using pricing rule (6) that uses constant markups, this system has at most six distinct prices (domestic and export price for normal city, same for the disturbed city, and for new town):

$$p_{ii} = \frac{\sigma c}{\sigma - 1},\ p_{i'i} = \frac{\sigma \tau c}{\sigma - 1},\ p_{nn} = \frac{\sigma c w_n}{\sigma - 1},$$

$$p_{i'n} = \frac{\sigma \tau c w_n}{\sigma - 1},\ p_{n+1,n+1} = \frac{\sigma c w_{n+1}}{\sigma - 1}, \quad p_{i',n+1} = \frac{\sigma \tau c w_{n+1}}{\sigma - 1}. \tag{15}$$

This system can be aggregated into price indexes, evaluated at point $\varepsilon = 0$, so that we distinguish only "big" cities from "new" one (by symmetry, we obtain $w_n = 1$):

$$P_i = \left[\frac{N\theta(N)}{F\sigma} \left(\frac{\sigma c}{\sigma - 1} \right)^{1-\sigma} (1 + (n-1)\tau^{1-\sigma}) \right]^{1/(1-\sigma)} \tag{16}$$

$$P_{n+1} = \left[\frac{N\theta(N)}{F\sigma} \left(\frac{\sigma \tau c}{\sigma - 1} \right)^{1-\sigma} n \right]^{1/(1-\sigma)} \tag{17}$$

At trade equilibrium the utilities depend also on wages. To find the wages we recall constant firm size and use market clearing equations (8) for varieties produced in n big cities and in $(n+1)$-st city which is small:

$$\frac{(1+(n-1)\tau^{1-\sigma})\theta(N)N[\sigma c/(\sigma-1)]^{-\sigma}}{P_i^{1-\sigma}} = (\sigma-1)F/c \tag{18}$$

$$\frac{n\tau\theta(N)N[\sigma\tau c w_{n+1}/(\sigma-1)]^{-\sigma}}{P_i^{1-\sigma}} = (\sigma-1)F/c. \tag{19}$$

Taking a ratio of these two conditions, we obtain the equilibrium (shadow) wage in $(n+1)$-st city:

$$w_{n+1}^{-\sigma} = \frac{1+(n-1)\tau^{1-\sigma}}{n\tau^{1-\sigma}} \tag{20}$$

Recall that $\theta(0)=1$. Therefore, we can rewrite comparison of utilities in small and big cities $V_{n+1} \leq V_n$ as comparison of real incomes $w_{n+1}/P_{n+1} \leq \theta(N)/P_n$. Substituting definition of $\theta(N)$, w_{n+1} and the ratio of price indexes from above we get result (9).

(2) Consider a city system $(n,\ N+\varepsilon, N-\varepsilon, N, ..., N)$ perturbed by small migration ε from the second city to the first one (as in definition of agglomeration stability). We apply the same method. There are again at most six distinct prices and we can write down the equilibrium equations for the price indexes and wages taking labor in cities $i=\overline{3,n}$ as numeraire:

$$P_1 = \left[\frac{(N+\varepsilon)\theta(N+\varepsilon)}{F\sigma}\left(\frac{\sigma c w_1}{\sigma-1}\right)^{1-\sigma} + \frac{(N-\varepsilon)\theta(N-\varepsilon)}{F\sigma}\left(\frac{\sigma\tau c w_2}{\sigma-1}\right)^{1-\sigma} + \frac{N\theta(N)}{F\sigma}\left(\frac{\sigma\tau c}{\sigma-1}\right)^{1-\sigma}(n-2)\right]^{1/(1-\sigma)} \tag{21}$$

$$P_2 = \left[\frac{(N+\varepsilon)\theta(N+\varepsilon)}{F\sigma}\left(\frac{\sigma\tau c w_1}{\sigma-1}\right)^{1-\sigma} + \frac{(N-\varepsilon)\theta(N-\varepsilon)}{F\sigma}\left(\frac{\sigma c w_2}{\sigma-1}\right)^{1-\sigma} + \frac{N\theta(N)}{F\sigma}\left(\frac{\sigma\tau c}{\sigma-1}\right)^{1-\sigma}(n-2)\right]^{1/(1-\sigma)}$$

$$P_i = \left[\frac{(N+\varepsilon)\theta(N+\varepsilon)}{F\sigma}\left(\frac{\sigma\tau c w_1}{\sigma-1}\right)^{1-\sigma} + \frac{(N-\varepsilon)\theta(N-\varepsilon)}{F\sigma}\left(\frac{\sigma\tau c w_2}{\sigma-1}\right)^{1-\sigma} + \frac{N\theta(N)}{F\sigma}\left(\frac{\sigma c}{\sigma-1}\right)^{1-\sigma}(1+(n-3)\tau^{1-\sigma})\right]^{1/(1-\sigma)}$$

$$\frac{(N+\varepsilon)\theta(N+\varepsilon)w_1[\sigma c w_1/(\sigma-1)]^{-\sigma}}{P_1^{1-\sigma}} + \frac{\tau(N-\varepsilon)\theta(N-\varepsilon)w_2[\sigma\tau c w_1/(\sigma-1)]^{-\sigma}}{P_2^{1-\sigma}} + (n-2)\frac{\tau N\theta(N)[\sigma\tau c w_1/(\sigma-1)]^{-\sigma}}{P_i^{1-\sigma}} = (\sigma-1)F/c \tag{22}$$

$$\frac{\tau(N+\varepsilon)\theta(N+\varepsilon)w_1[\sigma\tau c w_2/(\sigma-1)]^{-\sigma}}{P_1^{1-\sigma}} + \frac{(N-\varepsilon)\theta(N-\varepsilon)w_2[\sigma c w_2/(\sigma-1)]^{-\sigma}}{P_2^{1-\sigma}}$$
$$+(n-2)\frac{\tau N\theta(N)[\sigma\tau c w_2/(\sigma-1)]^{-\sigma}}{P_i^{1-\sigma}} = (\sigma-1)F/c$$

Differentiating this system of equation w.r.t. ε we aim to sign $\frac{dV_1}{d\varepsilon}$ at the symmetric point $\varepsilon = 0$. Note that: (1) at the symmetric point $w_1 = w_2 = 1$; (2) by symmetry and definition of N_2 we have $\frac{dw_2}{d\varepsilon} = -\frac{dw_1}{d\varepsilon}$ and similar equality applies to price indexes; finally, $\frac{dP_i}{d\varepsilon} = 0$, i.e. for any third city ($i \neq 1,2$) the effect from population increase in city #2 cancels out with the effect from exactly same decrease in city #1. Denote the elasticity of any variable X with respect to ε as $\mathcal{E}^X \equiv \frac{dX}{d\varepsilon}\frac{\varepsilon}{X}$ and totally differentiate equations (21) and (22) with respect to ε we obtain:

$$(1-\sigma)(1+(n-1)\tau^{1-\sigma})\mathcal{E}^{P_1} = (1-\tau^{1-\sigma})(1+\mathcal{E}^\theta + (1-\sigma)\mathcal{E}^{w_1}) \tag{23}$$

$$\sigma(1+(n-1)\tau^{1-\sigma})\mathcal{E}^{w_1} = (1-\tau^{1-\sigma})(1+\mathcal{E}^\theta + \mathcal{E}^{w_1} + (\sigma-1)\mathcal{E}^{P_1}) \tag{24}$$

We are interested in the elasticity of indirect utility, which can be expressed as $\mathcal{E}^{V_1} = \mathcal{E}^\theta + \mathcal{E}^{w_1} - \mathcal{E}^{P_1}$. Solution to the system of elasticity equations delivers:

$$\mathcal{E}^{w_1} - \mathcal{E}^{P_1} = (1+\mathcal{E}^\theta)\frac{2\sigma-1}{(\sigma-1)\left(\sigma-1+\sigma\frac{1+(n-1)\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\right)}$$

Therefore, stability condition $\mathcal{E}^{V_1} \leq 0$ can be rewritten in a form

$$\frac{2\sigma-1}{(\sigma-1)\left(\sigma-1+\sigma\frac{1+(n-1)\tau^{1-\sigma}}{1-\tau^{1-\sigma}}\right)} \leq -\frac{\mathcal{E}^\theta}{1+\mathcal{E}^\theta}.$$

Recall that $\theta(N) = 1-k\sqrt{N}$ and, hence, $\mathcal{E}^\theta = -\frac{k\sqrt{N}}{2(1-k\sqrt{N})}$. Substituting $\mathcal{E}^\theta$ into previous inequality delivers result (10). Q.E.D.

Proof of Proposition 2. As we have shown in the proof of Proposition 1, critical $L^d(n)$ approaches zero with a speed $1/n$. Therefore, consider the following limit and apply to it l'Hospital's rule:

$$\lim_{n\to\infty}\left[k\sqrt{nL^d(n)} = \frac{1-\left(1+\frac{1-\phi}{n\phi}\right)^{-\frac{2\sigma-1}{(\sigma-1)\sigma}}}{1/n} = \frac{\frac{2\sigma-1}{\sigma(\sigma-1)}\left(1+\frac{1-\phi}{n\phi}\right)^{-\frac{2\sigma-1}{(\sigma-1)\sigma}-1}\left(-\frac{1-\phi}{n^2\phi}\right)}{-\frac{1}{n^2}}\right] = \frac{(2\sigma-1)(1-\phi)}{\sigma(\sigma-1)\phi}$$

Similarly,

$$\lim_{n\to\infty}\left[k\sqrt{nL^a(n)} = \frac{\frac{2}{\sigma+2+\frac{(\sigma-1)\sigma n\phi}{(2\sigma-1)(1-\phi)}}}{1/n} = \frac{2n}{\sigma+2+\frac{(\sigma-1)\sigma n\phi}{(2\sigma-1)(1-\phi)}}\right] = \frac{2(2\sigma-1)(1-\phi)}{\sigma(\sigma-1)\phi}$$

Combining these limits together and applying continuity we obtain $\lim_{n\to\infty} n(L^a(n)-L^d(n)) > 0$. This implies that there exists $\bar{n}$ such that $\forall n \geq \bar{n} \quad L^a(n) > L^d(n)$, or equivalently, there is no

L such that $L^a(n) \leq L \leq L^d(n)$, which is the necessary condition for migration stable equilibrium. Q.E.D.

Proof of Proposition 5. Let as perform comparative static exercise w.r.t. N_1. Taking elasticity of the equilibrium conditions we find

$$(1-\sigma)(1+(n-1)\tau^{1-\sigma})\mathcal{E}_d^{P_n} = 1+\mathcal{E}^\theta + (1-\sigma)\mathcal{E}_d^{w_n} \tag{25}$$

$$\sigma(1+(n-1)\tau^{1-\sigma})\mathcal{E}_d^{w_n} = 1+\mathcal{E}^\theta + \mathcal{E}_d^{w_n} + (\sigma-1)\mathcal{E}_d^{P_n} \tag{26}$$

Here the subscript d emphasizes that elasticity is perceived by developer. They look very much alike the elasticity equations for the stability against agglomeration (23) and (24), only without $(1-\phi)$ multiplier on the right hand side. Therefore, the costlier the trade is, the more developer stability behavior resembles that of stability against agglomeration. The intuition is straightforward: cities affect each other through trade only. Therefore, the higher are trade cost, the less impact developer's city has on other cities, and therefore, the smaller is the developer's mistake in assumption of no change in other cities. Solving these equations and evaluating the elasticity of indirect utility we obtain:

$$\mathcal{E}_d^{V_1} = \frac{2\sigma-1}{(\sigma-1)(2\sigma-1+\sigma(n-1)\phi)}(1+\mathcal{E}^\theta)+\mathcal{E}^\theta \tag{27}$$

Straightforward algebra yields the result.

B Extension to two sectors

A thoughtful reader could noticed the discrepancy between our theoretical framework and its empirical interpretation. Describing "deurbanization" of a particular manufacturing industry among others, we have dealt so far with general equilibrium model with one sector only. However, this section shows how our setup may be embedded into multi-sector framework maintaining qualitatively similar results.

Model. Assume our small-city system remains the same. However, now it trades also with outside world that produces some aggregate good M (money), which describes, as usual, other sectors. Our citizens consume two goods: composite good U produced in our cities (defined as 2) and good M produced outside. Preferences over the two goods have standard Cobb-Douglas form:

$$\check{U} = \left(\left[\sum_{i=1}^{n}\int_0^{m_i} x_{kji}^{(\sigma-1)/\sigma}dj\right]^{\sigma/(\sigma-1)}\right)^{\mu} M^{1-\mu}$$

where $\check{U}$ denotes overall utility of consumption and $0<\mu<1$. It is maximized under the budget constraint

$$\sum_{i=1}^{n}\int_0^{m_i} p_{kji}x_{kji}dj + P_M M \leq I(N_k)$$

that includes outside good and its price P_M, that may include cost of transportation to our cities. Internal structure of our city system remains the same, however, now cities trade their local good for outside one. Outside world consists of some L_2 consumers having similar utilities and producing outside good M. Our firms and developers take the price P_M of outside world as given either because this world is big enough or it exploits linear technology.

Analysis. Under Cobb-Douglas preferences, the indirect utility in any city i is defined (up to a constant multiplier) as follows:

$$V_i = \frac{\theta(N_i)w_i}{P_i^{\mu} P_M^{1-\mu}}.$$

One can see that new indirect utility has the *only* difference from previous V_i: it has power $\mu < 1$ over the local good price index P_i. All our results are derived from such indirect function (it summarizes other elements of the model). Hence, to be sure that our results apply in the extended version of the model it is sufficient to discuss: What changes when μ becomes smaller than 1?

Presence of the outside good *softens* the impact of local price index on utility. Does this fact work in favor or against agglomeration? Other things equal, one of positive effects of growing city is decreasing price index, being composed of more cheaper home-produced varieties relative to more expensive imported ones. This price effect acts towards agglomeration. Therefore, among systems with different μ, the one described in details before ($\mu = 1$) has the greatest driving force for agglomeration. Thereby, our dispersion results for $\mu = 1$ remain valid for any smaller μ, even being enforced (the zone of stable equilibria must shrink). This argumentation bridges our economic interpretation with the developed theoretical setup.

Interpretation. In comparing the model with reality, the comparative statics in μ suggests a testable prediction. Indeed, under Cobb-Douglas preferences, parameter μ represents the share of income spent on the local good. Recall also (see Fig.3 and related discussion) that the cities' size decreases in response to decreasing trade costs, or growing population. A new topic is comparative statics in parameter μ. It should push cities' size in the same direction as trade freeness ϕ, because both work as dispersion forces (more accurate proof needs further study), against agglomeration. Thus, in cross-section comparison among industries, one would expect to observe *negative* correlation between the urbanization level of each sector and the share of consumer spending on its product. Implementing this task, one needs to control only for technological difference in transportation and employment. Cost parameters (c, F) do not affect the city size. To the best of our knowledge, this hypothesis is novel, and it can be tested in future work.